EMC Course Notes 2024

EM Coupling Mechanisms

Common Impedance Coupling

If you've ever noticed the lights flicker when a large home appliance kicks on, you have some experience with common impedance coupling. Common impedance coupling (also called conducted coupling) may occur anytime a source circuit and a victim circuit share part of their respective current paths.

Consider the two simple circuits shown in Figure 3.1. Each circuit has its own source, signal wire and load, but they share a wire for the signal return current. If the shared wire had zero impedance, the voltage across each circuit's load resistor would depend only on that circuit's source voltage. However, a small amount of impedance in the shared wire causes a voltage to appear across R_{L2} when there is a signal in Circuit 1 and vice versa.

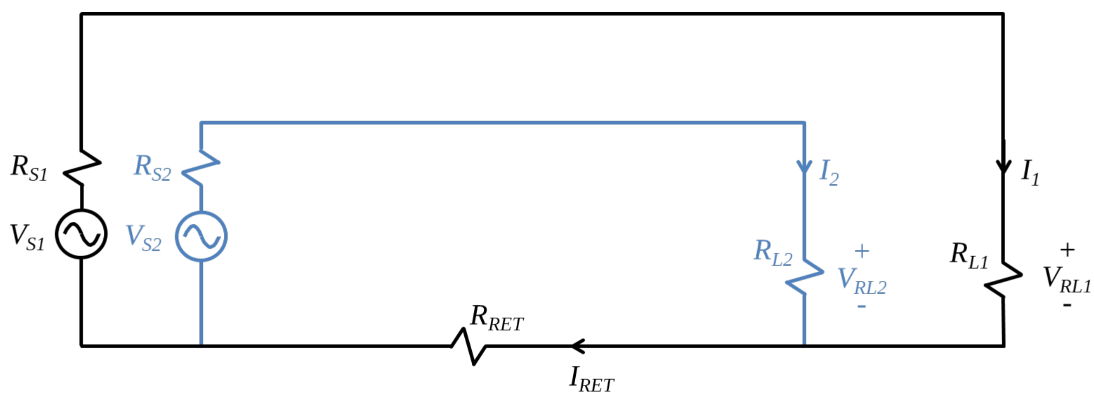


Figure 3.1. Two circuits sharing a common signal return.

This phenomenon is called *crosstalk*. Crosstalk is a term commonly used to describe the unintentional electromagnetic coupling between two circuits or systems in close proximity. Although there is no accepted standard for quantifying crosstalk, it is often expressed as a ratio of the coupled voltage difference in the victim circuit to the signal voltage in the source circuit. In this case, we'll calculate the crosstalk as,

$$\text{crosstalk in dB} = 20\log\left|\frac{\text{coupled voltage appearing at receiver in Circuit 2}}{\text{signal voltage in Circuit 1}}\right| \tag{3.1}$$

or,

$$\text{Xtalk}_{21} = 20\log\left|\frac{V_{RL2}}{V_{RL1}}\right|_{when\ V_{S2}=0} . \tag{3.2}$$

To calculate the crosstalk in Circuit 2 due to the signals in Circuit 1, we set $V_{S2} = 0$ and determine the ratio V_{RL2}/V_{RL1}. Applying Kirchhoff's voltage law to the Circuit 2 current loop, we have

$$V_{S2} + I_2 R_{S2} + I_2 R_{L2} + \left(I_1 + I_2\right) R_{RET} = 0 . \tag{3.3}$$

Setting $V_{S2} = 0$, we can rewrite Equation (3.3) to express I_2 in terms of I_1,

$$I_2 = \frac{-R_{RET}}{R_{S2} + R_{L2} + R_{RET}} I_1 . \tag{3.4}$$

Noting that $I_1 = V_{RL1}/R_{L1}$ and $I_2 = V_{RL2}/R_{L2}$, we can express (3.4) in terms of the voltages across the load resistances,

$$\frac{V_{RL2}}{R_{L2}} = \frac{-R_{RET}}{R_{S2} + R_{L2} + R_{RET}} \frac{V_{RL1}}{R_{L1}} . \tag{3.5}$$

The crosstalk can now be expressed as,

$$20 \log \left| \frac{V_{RL2}}{V_{RL1}} \right| = 20 \log \left| \frac{R_{RET}}{R_{S2} + R_{L2} + R_{RET}} \frac{R_{L2}}{R_{L1}} \right| . \tag{3.6}$$

Example 3-1: Calculating the crosstalk between two 50-Ω circuits

For the circuit in Figure 3.1, let $R_{S1} = R_{L1} = R_{S2} = R_{L2} = 50\ \Omega$ and assume the resistance of the signal return wire is 100 mΩ. The crosstalk is then,

$$\text{Xtalk}_{21} = 20 \log \left[\frac{0.1}{50 + 50 + 0.1} \left(\frac{50}{50} \right) \right] = -60 \text{ dB} .$$

In other words, a signal voltage of 5 volts in Circuit 1 creates 5 mV of noise in Circuit 2.

Generally, for practical common impedance coupling situations, the impedance of the common return path will be much less than the load impedances of either circuit. Otherwise, the return path would severely attenuate the signals. In most cases, it is much quicker to estimate the voltage dropped across the signal return path as,

$$V_{RET} = I_{signal} R_{RET} \ . \tag{3.7}$$

In the case above, I_{signal} was I_1, the intentional current. Once we have estimated V_{RET}, we can determine what fraction of this voltage difference appears across the victim circuit's load resistance. In this case,

$$\begin{aligned} V_{RL2} &\approx \frac{R_{L2}}{R_{L2} + R_{S2}} V_{RET} \\ &\approx \frac{R_{L2}}{R_{L2} + R_{S2}} R_{RET} I_1 \\ &\approx \frac{R_{L2}}{R_{L2} + R_{S2}} R_{RET} \frac{V_{RL1}}{R_{L1}} \end{aligned} \tag{3.8}$$

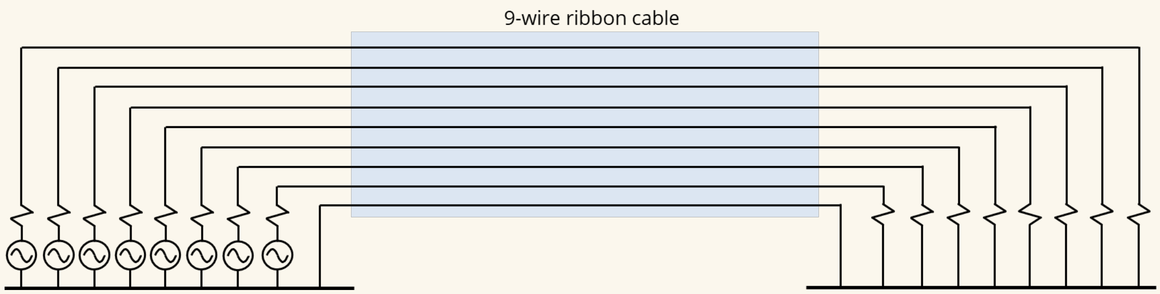
and the crosstalk is,

$$\text{Xtalk}_{21} \approx 20 \log \left[\frac{R_{RET}}{R_{L2} + R_{S2}} \left(\frac{R_{L2}}{R_{L1}} \right) \right] . \tag{3.9}$$

This approach generally yields very good estimates of the crosstalk and is much simpler to apply to complex configurations with many possible signal return paths.

Example 3-2: Common Impedance Coupling in a Ribbon Cable

The 20-cm, 9-wire ribbon cable illustrated in the figure below carries an 8-bit data bus (8 data wires and 1 return wire). The resistance per unit length of each wire in the cable is 1.1 Ω/m at 2.0 MHz. If each of the data lines is driven by a 10-Ω source and terminated with a 50-Ω resistance to the common return wire, calculate the crosstalk between any two data lines that is due to common impedance coupling at 2.0 MHz.



An 8-bit data bus on a 9-wire ribbon cable

Due to the symmetry of this problem, we can start by turning any one of the sources on and the others off. In this case, the current in the source circuit will be,

$$I_1 = \frac{V_{RL1}}{R_{L1}}.$$

After flowing through the load resistor, R_{L1}, the current can return to the source through the ribbon cable resistance or through the other 7 load and source resistances. The resistance of the return wire is,

$$R_{wire} = 0.2 \text{ m} \times 1.1 \ \Omega/\text{m} = 0.22 \ \Omega.$$

The path resistance through the other seven source and load resistances in parallel is,

$$R_{2-7} = \frac{R_S + R_L + R_{wire}}{7} = \frac{10 + 50 + 0.22}{7} = 8.6 \ \Omega.$$

Therefore, nearly all (98%) of the source current will return through the return conductor in the ribbon cable. The voltage dropped across the return conductor will then be,

$$V_{RET} = I_1 R_{wire}.$$

By voltage division, the voltage difference appearing across any of the other load resistors will be,

$$V_{RL2-7} \approx \frac{50}{10+50} V_{RET} = 0.18 \, I_1$$

and, therefore, the crosstalk can be expressed as,

$$\text{Xtalk} = 20\log\left[\frac{V_{RL2-7}}{V_{R1}}\right] = 20\log\left[\frac{0.18I_1}{50I_1}\right] = -49\text{ dB}.$$

If the 9-wire ribbon cable is replaced with a 10-wire ribbon cable and the extra wire is used as an additional signal return, the resistance of the common return path is cut in half. This would reduce the crosstalk by a factor of 2 (i.e., 6 dB). In this case the total crosstalk would be -49 – 6 = -55 dB. Providing additional return conductors would reduce the crosstalk even more.

Note that the frequency of the signal never appeared in this crosstalk calculation. Common impedance coupling is independent of frequency except to the extent that the shared impedance is frequency dependent. At low frequencies (kHz and lower), the impedance of the wire would be independent of the frequency and so would the level of crosstalk. At high frequencies (MHz and higher), the resistance of the wire would be proportional to the square root of the frequency due to the skin effect. In this case, the crosstalk would also increase as the square root of the frequency. However, as we will see in the next section, common impedance coupling is not likely to be the dominant coupling mechanism at high frequencies.

Electric Field Coupling

Electric field coupling (also called capacitive coupling) occurs when energy is coupled from one circuit to another through an electric field. As we shall see, this is most likely to happen when the impedance of the source circuit is high.

Consider the two circuits sharing a common return plane shown in Figure 3.2. If the return plane had zero resistance, the common impedance coupling would be zero. However, it is also possible for coupling to occur between the two circuits due to the electric field lines that start on one signal wire and terminate on the other. For example, if one of the signal voltages is +1 volt and the other is 0 volts, then the potential difference between the two signal wires results in electric field lines that start on the +1-volt wire and terminate on the 0-volt wire. Schematically, this can be represented by a capacitor between the two signal wires.

Of course, there are other electric field lines that start on the +1-volt wire and terminate on the 0-volt plane. This can be represented by a capacitance between the wire and the plane. A schematic representation of the two circuits in Figure 3.1 that includes the electric field coupling capacitances is shown in Figure 3.3.

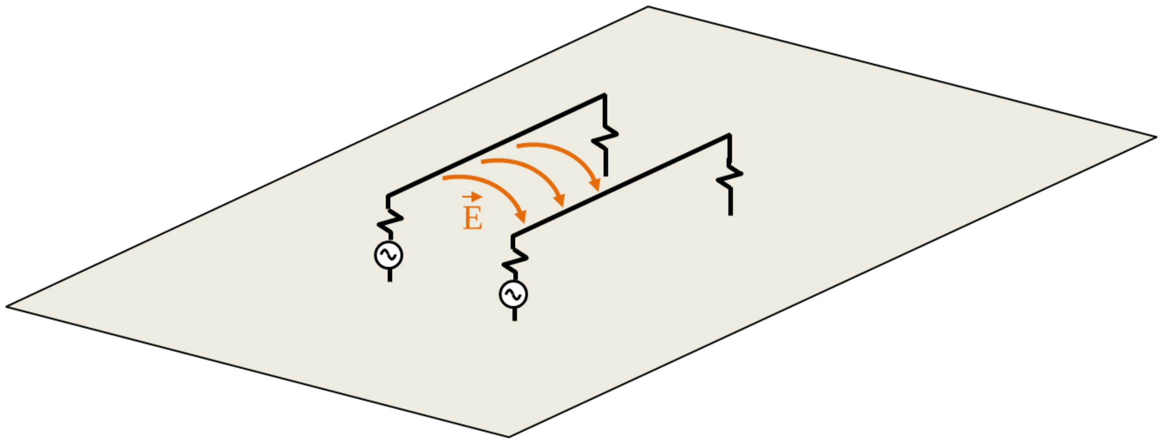


Figure 3.2. Two circuits above a signal return plane.

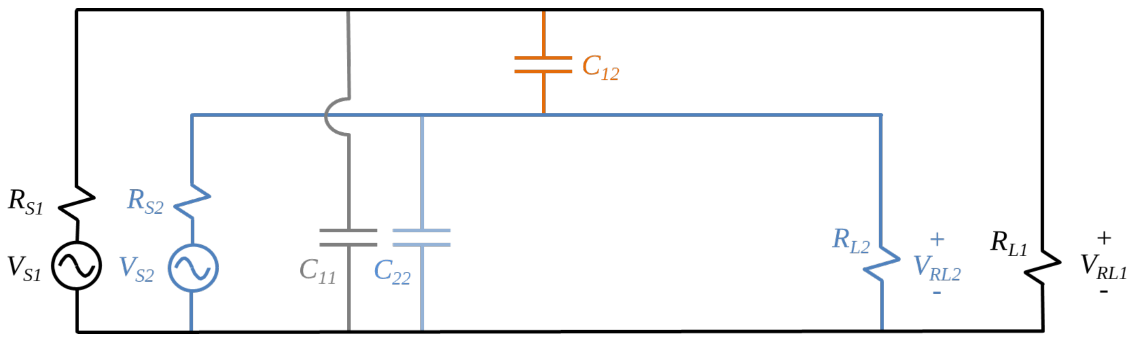


Figure 3.3. Schematic representation of the circuits in Figure 3.2 including capacitive coupling paths.

In this geometry, the plane is our zero-volt reference. C_{12} is the mutual capacitance between the wires. C_{11} and C_{22} are the capacitances of each wire to the plane. These capacitances can be determined using a 2D field solver or approximated using closed-form equations. Once the capacitances have been determined, and values have been assigned to all the elements in Figure 3.3, the crosstalk due to electric field coupling can be calculated using the same basic formula used for common impedance coupling,

$$\text{Xtalk}_{21} = 20 \log \left| \frac{V_{RL2}}{V_{RL1}} \right|_{when\ V_{S2}=0} . \tag{3.10}$$

If we try to find the exact solution, the procedure for analyzing this circuit with 9 elements can be time consuming. However, if we redraw the circuit and take advantage of the relative size of some of the impedances, we can greatly simplify the analysis.

First, let's redraw the circuit in Figure 3.3 as shown in Figure 3.4. By putting Circuit 1 on the left side of the schematic and Circuit 2 on the right side, the important coupling, C_{12}, is clearer. Also, it is helpful to recognize that the impedances of the self-capacitances C_{11} and C_{22} are almost always much higher than the load impedances. If this were not true, the signal reaching the load would be significantly attenuated. Therefore, we can usually neglect C_{11} and C_{22} when solving circuits like the one in Figure 3.4.

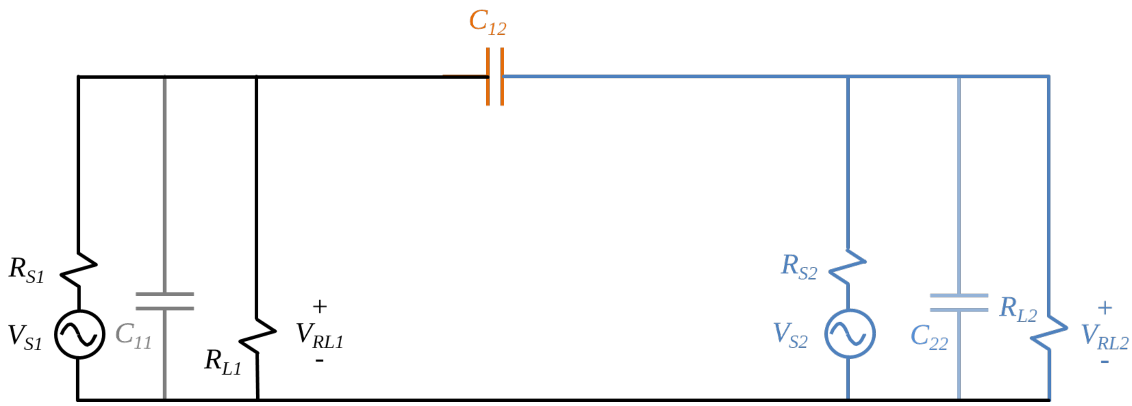


Figure 3.4. More intuitive schematic representation of the circuits in Figure 3.3.

To calculate the crosstalk in Circuit 2 due to the signals in Circuit 1, we set $V_{S2} = 0$ and determine the ratio V_{RL2}/V_{RL1}. If the coupling is relatively weak (i.e., if the coupling is not loading down the source circuit), then the impedance of C_{12} is large relative to the impedances in Circuit 1. This means the value of V_{RL1} is independent of the Circuit 2 parameters and the circuit can be represented in the simple form shown in Figure 3.5.

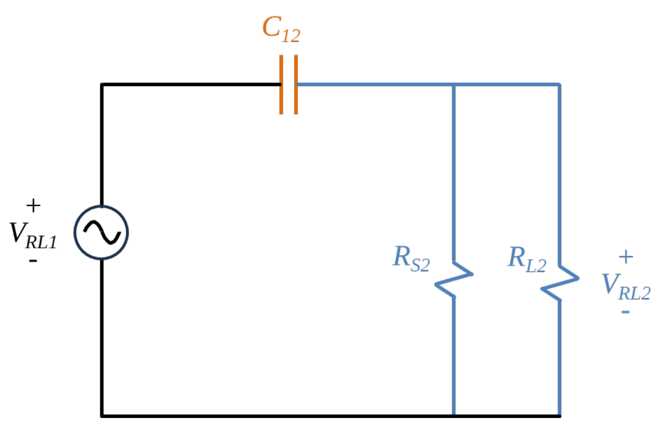


Figure 3.5. An even simpler representation of the circuits in Figure 3.3.

Now the circuit is relatively easy to solve. The crosstalk can be expressed as,

$$20\log\left|\frac{V_{RL2}}{V_{RL1}}\right| = 20\log\left|\frac{R_{S2}\,||\,R_{L2}}{R_{S2}\,||\,R_{L2} + \left(1/j\omega C_{12}\right)}\right|. \tag{3.11}$$

Example 3-3: Calculating the crosstalk between two 150-Ω circuits

For the circuits in Figures 3.2 and 3.3, assume the signal wires are 4.0 mm above the conducting plane and 16 cm long. Assume the wire radius is 0.8 mm and the spacing between the wires is 3.0 mm. Let $R_{S1} = R_{S2} = 10\ \Omega$ and $R_{L1} = R_{L2} = 150\ \Omega$. Calculate the crosstalk due to electric field coupling between these circuits at 50 MHz.

We start by determining the capacitances C_{11}, C_{22} and C_{12}. The capacitance of each wire above the plane is approximately,

$$C_{11} = C_{22} = \frac{(0.16)2\pi\varepsilon_0}{\cosh^{-1}\left(4.0/0.8\right)} = 3.8 \text{ pF.}$$

The capacitance between the two wires is approximately,

$$C_{12} = \frac{(0.16)\pi\varepsilon_0}{\cosh^{-1}\left(3.0/1.6\right)} = 3.6 \text{ pF.}$$

The impedance of C_{11} and C_{22} at 50 MHz is $|1/j\omega C| = 800\ \Omega$. Since this is much higher than the 150-Ω circuit impedance, we can neglect these capacitances. The impedance of the coupling capacitance is $|1/j\omega C| = 890\ \Omega$. This is also much greater than the circuit impedances, therefore we can use Equation (3.11) to calculate the crosstalk,

$$\text{Xtalk}_{21} = 20\log\left|\frac{V_{RL2}}{V_{RL1}}\right| = 20\log\left|\frac{10\,||\,150}{10\,||\,150 + j890}\right| = -40 \text{ dB.}$$

It is helpful to note how changing the various circuit parameters would have changed the coupling in this case. For example, doubling the frequency would have doubled the crosstalk (i.e., at 100 MHz, the calculated crosstalk would be -34 dB). For the weak coupling case, electric field coupling is proportional to frequency.

Doubling the source resistance of the victim circuit would also have doubled the crosstalk in this example. Note that the parallel combination of the source and load resistances was almost equal to the source resistance. In this example, doubling the load resistance would have little effect on the crosstalk, since it is the parallel combination of the source and load resistances in the victim circuit that is important.

The other important parameter in this example is the mutual capacitance, C_{12}. Reducing the value of C_{12} would reduce the crosstalk proportionally. Moving the wires farther apart is one way to reduce the value of C_{12}. However, it is important to note that merely doubling the distance between the wires is not sufficient to reduce C_{12} by a factor of 2. The inverse hyperbolic cosine function behaves like a logarithmic function when the wire separation is greater than the wire diameter. In this case, doubling the distance between the wires (from 3.0 mm to 6.0 mm) would have changed the value of C_{12} from 3.6 pF to 2.2 pF. This would have reduced the crosstalk by only about 4 dB.

Faraday's Law

In the early 1800s, an Englishman named Michael Faraday discovered that a time-varying magnetic field induced a voltage difference in an electric circuit. This discovery was an important step toward the development of electromagnetic theory and *Faraday's Law* in its various forms continues to be one of the more important equations for EMC engineers today.

Quiz Question:

The algebraic sum of all the voltages around any closed path in a circuit equals zero.

a. true
b. false

Electrical engineers will recognize the sentence above as a statement of Kirchhoff's Voltage Law (KVL). Nevertheless, if we are talking about real implementations of circuits, the correct answer is *false*. KVL is a very useful concept and a cornerstone of circuit theory. Unfortunately, circuit theory can't always be applied to the real implementation of a circuit.

Unlike Maxwell's equations, which must be satisfied in any situation, the equations of circuit theory rely on a variety of simplifying assumptions about the nature of circuits and components. For example, let's take a closer look at Faraday's Law, which is one of Maxwell's equations,

$$\oint \vec{E} \cdot \vec{d\ell} = -\frac{\partial}{\partial t} \int_s \vec{B} \cdot \vec{ds} \, . \tag{3.12}$$

The equation basically says that if we were to define a closed path in any arbitrary location and integrate the electric field around that loop, the total voltage obtained would be equal to the time-rate-of-change of the magnetic flux passing through that closed path.

This equation always holds no matter how or where we define the path, so let's apply Faraday's Law to the circuit in Figure 3.6.

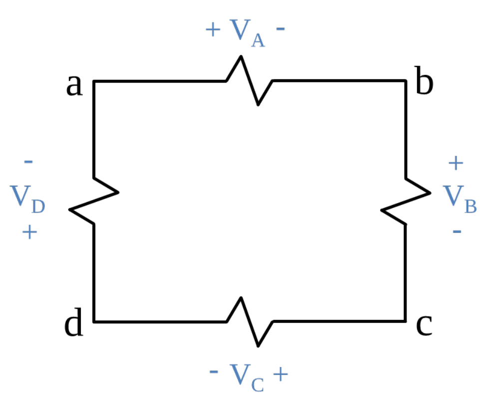


Figure 3.6. Circuit with 4 resistors.

The circuit consists of four resistors connected in a loop using perfectly conducting wire. We can choose our path to be anywhere, so let's define it to be along the center of the wire and through the middle of each resistor. The electric field inside a perfectly conducting wire must be zero; therefore, in this case, Faraday's Law (3.12) can be simplified to,

$$\int_a^b \vec{E}\cdot\overrightarrow{d\ell} + \int_b^c \vec{E}\cdot\overrightarrow{d\ell} + \int_c^d \vec{E}\cdot\overrightarrow{d\ell} + \int_d^a \vec{E}\cdot\overrightarrow{d\ell} = -\frac{\partial}{\partial t}\oint_S \vec{B}\cdot\overrightarrow{ds}\,. \tag{3.13}$$

By definition, $\int_a^b \vec{E}\cdot\overrightarrow{d\ell}$ is the voltage difference between *a* and *b*, or in this example, the voltage across the top resistor, V_A. The other terms in (3.13) are the voltages dropped across the other resistors in the circuit. Therefore (3.13) can be written,

$$V_A + V_B + V_C + V_D = -\frac{\partial}{\partial t}\oint_S \vec{B}\cdot\overrightarrow{ds}\,. \tag{3.14}$$

If there are no time-varying quantities (i.e., the magnetic flux density is constant and the circuit doesn't move), then the right-hand side of (3.14) is zero and Faraday's Law can be written as,

$$V_A + V_B + V_C + V_D = 0\,. \tag{3.15}$$

This is KVL for this circuit.

However, if the flux and/or the circuit change with time, then the right-hand side of (3.14) is not zero and KVL does not apply. The term on the right-hand side of (3.14) represents the time-rate-of-change of the total magnetic flux passing through the circuit. The total flux is the integral of the flux density over the circuit loop area,

$$\Psi = \oint_S \vec{B}\cdot\overrightarrow{ds}\,. \tag{3.16}$$

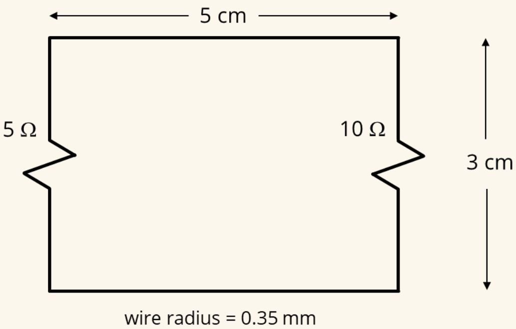
Therefore, for circuit loops of arbitrary shape and size consisting of small components connected by perfectly conducting wire, Faraday's Law tells us that,

$$\sum \text{voltages dropped across components in the loop} = -\frac{\partial \Psi}{\partial t}\,. \tag{3.17}$$

where Ψ is the total magnetic flux passing through the loop formed by the circuit elements. For convenience, we will refer to the sum of all voltages dropped across components in a loop as V_{LOOP}.

Example 3-4: High Impedance Loop in a Uniform H-Field

Consider the circuit below consisting of two resistors connected with wire forming a 5-cm by 3-cm loop. If the circuit is located in a 150 kHz, 2.0 Amp/m magnetic field, determine the voltage induced across the 10-Ω resistor. The direction of the magnetic field is perpendicular to the plane of the paper (i.e., maximum coupling).



The sum of the voltages across the two resistors in this circuit will equal the derivative of the total magnetic flux coupling the loop,

$$\left|V_{LOOP}\right| = \omega\Psi.$$

Since the field in the loop is uniform, the total flux Ψ is equal to the magnitude of the flux density, $\vec{B}$, times the loop area. The loop is in free space, so the flux density, $\vec{B}$, is equal to $\mu_0 \vec{H}$ and the loop voltage is,

$$\begin{aligned}\left|V_{LOOP}\right| &= \omega\Psi \\ &= \omega\mu_0 \left|\vec{H}\right| A \\ &= \left(2\pi\times 150\times 10^{3}\,\text{sec}^{-1}\right)\left(4\pi\times 10^{-7}\text{ H/m}\right)\left(2\text{ A/m}\right)\left(.05\times .03\,\text{m}^{2}\right) \\ &= 3.55\times 10^{-3}\text{ V.}\end{aligned}$$

The voltage dropped across the 10-Ω resistor is a fraction of the total voltage dropped across all components in the loop. Using voltage division, we can express the voltage dropped across the 10-Ω resistor as,

$$\left|V_{R10}\right| = \frac{10}{10+5}\left|V_{LOOP}\right| = 2.4\,\text{mV}.$$

Suppose the circuit in Example 3-4 had no resistors. If it were a perfectly conducting loop of wire, the E-field would have to be zero everywhere inside the wire and the value of $\oint \vec{E}\cdot\vec{d\ell}$ would be zero. According to Faraday's Law,

$$\oint \vec{E}\cdot\vec{d\ell} = -\frac{\partial\Psi}{\partial t} = 0\,. \qquad (3.18)$$

In other words, the net time-varying flux passing through a perfectly conducting loop must always equal zero. How can this be true?

If a perfectly conducting loop is placed in a time-varying magnetic field, current induced in the loop creates an opposing magnetic field such that the total magnetic flux passing through the loop is exactly zero,

$$\Psi_{total} = \Psi_{incident} + \Psi_{induced} = 0 . \tag{3.19}$$

The magnitude of the induced current is,

$$\begin{aligned} \left|I_{induced}\right| &= \left|\frac{\Psi_{induced}}{L_{LOOP}}\right| \\ &= \left|\frac{-\omega\Psi_{incident}}{\omega L_{Loop}}\right| \\ &= \left|\frac{V_{LOOP}}{\omega L_{LOOP}}\right| . \end{aligned} \tag{3.20}$$

If the wire loop has finite resistance, or if there are resistors in the loop, the current in the loop is,

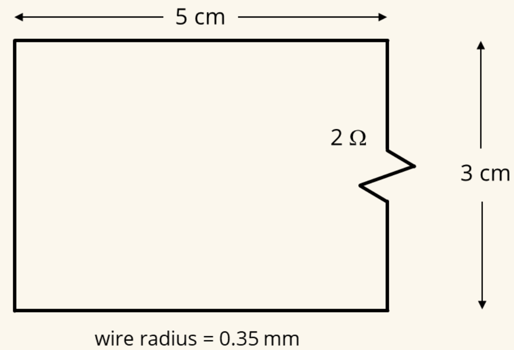
$$\left|I_{induced}\right| = \left|\frac{V_{LOOP}}{R_{LOOP} + j\omega L_{LOOP}}\right| , \tag{3.21}$$

where R_{LOOP} is the total loop resistance and L_{LOOP} is the total loop inductance. The voltage dropped across a small, lumped resistance, R_1, in the loop would be,

$$\left|V_{R1}\right| = \left|V_{LOOP}\right| \left|\frac{R_1}{R_{LOOP} + j\omega L_{LOOP}}\right| . \tag{3.22}$$

Example 3-5: Low Impedance Loop in a Uniform H-Field

Consider the circuit below consisting of a 2-Ω resistor connected to a 5-cm by 3-cm loop of wire. If the circuit is located in an 80-MHz, 500-μA/m magnetic field, determine the voltage induced across the 2-Ω resistor. The direction of the magnetic field is perpendicular to the plane of the paper (i.e., maximum coupling).



Using our equation for the inductance of a rectangular loop of wire, we can show that the inductance of this loop is 125 nH. At 80 MHz, the inductive reactance of the loop is then ωL = 63 Ω.

Clearly the loop inductance will limit the amount of current induced and therefore the voltage difference that can appear across the resistor. However, we can still calculate the quantity V_{LOOP} as follows,

$$\begin{aligned}\left|V_{LOOP}\right| &= \omega\Psi \\ &= \omega\mu_0\left|\vec{H}\right|A \\ &= \left(2\pi\times 80\times 10^6 \text{ sec}^{-1}\right)\left(4\pi\times 10^{-7} \text{ H/m}\right)\left(500\times 10^{-6} \text{ A/m}\right)\left(.05\times .03\,\text{m}^2\right) \\ &= 474\times 10^{-6} \text{ V.}\end{aligned}$$

The voltage dropped across the 2-Ω resistor can then be determined as a fraction of V_{LOOP} using (3.22),

$$\left|V_{R2}\right| = \left|474\times 10^{-6}\,\text{volts}\right|\left|\frac{2\,\Omega}{2\,\Omega + j63\,\Omega}\right| = 15\ \mu\text{V.}$$

Magnetic Field Coupling

Magnetic field coupling (also called inductive coupling) occurs when energy is coupled from one circuit to another through a magnetic field. Since currents are the sources of magnetic fields, this is most likely to happen when the impedance of the source circuit is low.

Consider the two circuits sharing a common return plane shown in Figure 3.7. Coupling between the circuits can occur when the magnetic field lines from one of the circuits pass through the loop formed by the other circuit. Schematically, this can be represented by a mutual inductance between the two signal wires as shown in Figure 3.8.

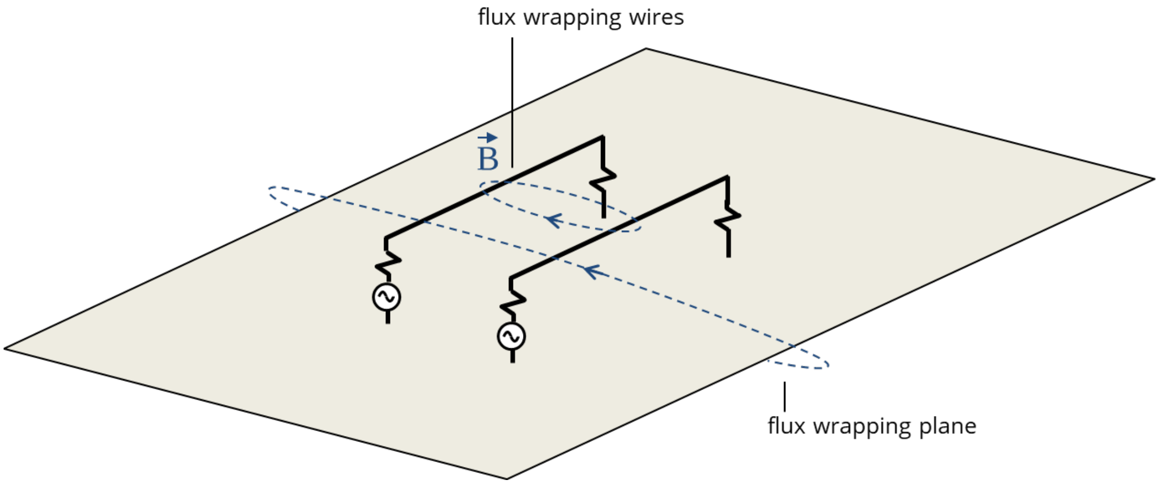


Figure 3.7. Two circuits above a signal return plane.

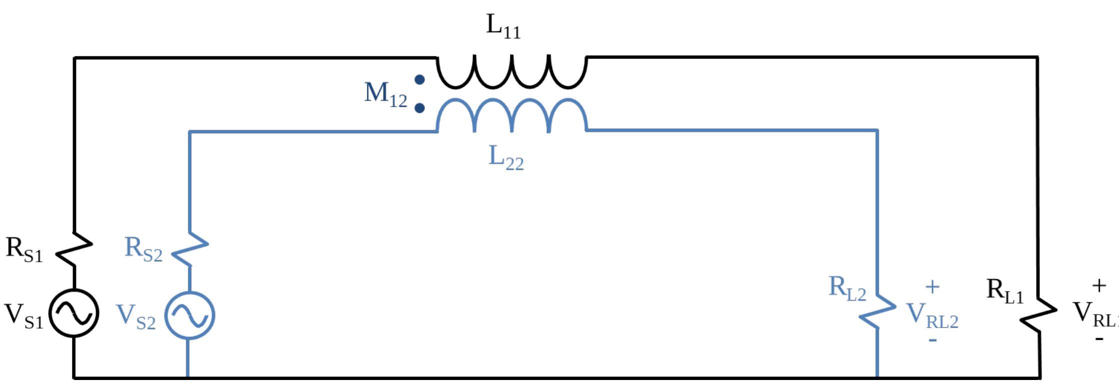


Figure 3.8. Schematic representation of the circuits in Figure 3.7 including inductive coupling.

In most cases, a convenient closed-form equation for calculating the mutual inductance will not be available. However, we can often estimate the mutual inductance by estimating the percentage of the total magnetic flux generated by the first loop that couples the second loop. For example, suppose the two wires in the example above are 20 mm above the plane and separated by 5 mm. We could visualize the magnetic flux lines that wrap the current in line 1 as shown in Figure 3.9.

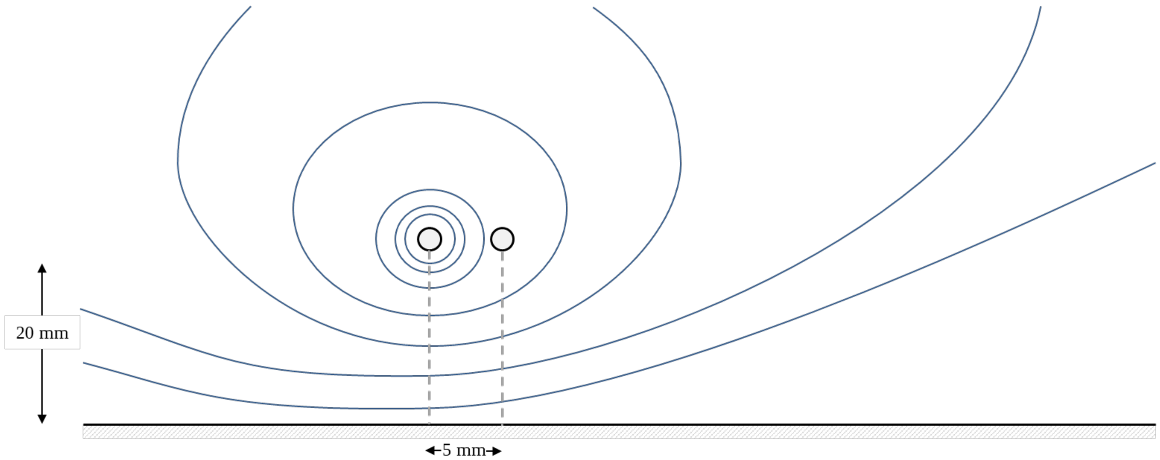


Figure 3.9. Magnetic flux lines coupling two circuits above a conducting plane.

If the wire radius in the example above is 0.6 mm, we could calculate the self inductance of the source circuit using the equation for the inductance per unit length of a wire over a conducting plane,

$$L_{11} \approx \frac{\mu_0}{2\pi} \cosh^{-1}\left(\frac{h}{a}\right) = 2\times 10^{-7} \cosh^{-1}\left(\frac{20}{0.6}\right) = 840 \text{ nH/m}. \tag{3.33}$$

The self inductance is the total flux divided by the current while the mutual inductance is the flux that couples both loops divided by the current. Therefore, the mutual inductance can be expressed as a fraction of the self inductance,

$$M_{12} = \left(\frac{\text{magnetic flux coupling both circuits}}{\text{total magnetic flux}}\right) L_{11}. \tag{3.34}$$

Based on the rough sketch in Figure 3.9, we might estimate that somewhere between 50% and 80% of the flux couples both circuits. If we were to assume 60%, then our estimate of the mutual inductance would be,

$$M_{12} \approx 0.6\, L_{11} = 500 \text{ nH/m}. \tag{3.35}$$

Of course, there are more accurate ways of determining the mutual inductance between two circuits. Electromagnetic modeling software is often used for this purpose when it is necessary to determine crosstalk levels more precisely. There are also a number of closed-form equations that can be applied to specific geometries. In fact, for the case of two thin wires above an infinite ground plane, there is a relatively simple closed form expression,

$$M_{12} = \frac{\mu_0}{4\pi} \ln\left(1 + 4\frac{h_1 h_2}{s^2}\right). \tag{3.36}$$

where h_1 and h_2 are the heights of the two wires above the plane, s is the distance between the two wires, and the wire radius is small relative to the height and separation. Applying this equation to the example above,

$$M_{12} = \frac{\mu_0}{4\pi} \ln\left(1 + 4\left(\frac{20}{5}\right)^2\right) = 420 \text{ nH/m.} \tag{3.37}$$

The difference between the estimate (3.35) and the calculation (3.37) is less than 2 dB. Estimates within a few dB are usually accurate enough to indicate whether a potential crosstalk problem exists.

To calculate the crosstalk due to magnetic field coupling, we start with the current in the source circuit, since the current is the source of the magnetic field. The voltage induced in the second circuit can be expressed as,

$$V_{LOOP2} = j\omega M_{12} I_1 . \tag{3.38}$$

V_{LOOP2} is the voltage induced in the entire loop of the circuit. The fraction of this voltage that will appear across the load can be expressed as,

$$V_{RL2} = V_{LOOP2} \left| \frac{R_{L2}}{R_{L2} + R_{S2} + j\omega L_{22}} \right| . \tag{3.39}$$

Since, $I_1 = V_{RL1}/R_{L1}$, the crosstalk due to magnetic field coupling can be expressed as,

$$\text{Xtalk}_{21} = 20 \log \left| \frac{V_{RL2}}{V_{RL1}} \right|_{when\ V_{S2}=0} = 20 \log \left| \frac{\omega M_{12}}{R_{L1}} \left(\frac{R_{L2}}{R_{L2} + R_{S2} + j\omega L_{22}} \right) \right| . \tag{3.40}$$

Example 3-6: Calculating the crosstalk between two 50-Ω circuits

For the circuit illustrated in Figures 3.7 and 3.8, assume the signal wires are 16 cm long. Assume the wire radius is 0.6 mm, the height is 20 mm, and the spacing between the wires is 5.0 mm as illustrated in Figure 3.9. Let $R_{S1} = R_{S2} = 10$ and $R_{L1} = R_{L2} = 50\ \Omega$. Calculate the crosstalk due to magnetic field coupling between these circuits at 10 MHz.

The inductance of each circuit and the mutual inductance between the two circuits per unit length are given in (3.33) and (3.37). Multiplying by the length of the circuit, we get

$$L_{11} = L_{22} = 840 \text{ nH/m} \times 0.16 \text{ m} = 130 \text{ nH}$$
$$M_{12} = 420 \text{ nH/m} \times 0.16 \text{ m} = 67 \text{ nH.}$$

The impedance of L_{11} and L_{22} at 10 MHz is $j\omega L = 8\ \Omega$, which is small relative to the resistance of each circuit, so we can ignore it. Substituting the circuit values into Equation (3.40) we get,

$$\text{Xtalk}_{21} = 20 \log \left| \frac{2\pi \times 10^7 \left(67 \times 10^{-9}\right)}{50} \left(\frac{50}{50+10} \right) \right| = -23 \text{ dB.}$$

It is helpful to observe how changing the various circuit parameters would have changed the coupling. For example, doubling the frequency would have doubled the crosstalk (i.e., at 20 MHz, the calculated crosstalk would be -17 dB). Both inductive coupling and capacitive coupling are proportional to frequency for the weak coupling case with resistive loads.

Doubling the load resistance of the source circuit would also have nearly cut the current in the source circuit in half, which would have reduced the crosstalk by 6 dB. In this example, doubling the load resistance of the victim circuit would have had relatively little effect on the crosstalk, since most of V_{LOOP2} was already dropped across the load.

The other important parameter in this example is the mutual inductance, M_{12}. Reducing the value of M_{12} would reduce the crosstalk proportionally. Moving the wires farther apart is one way to reduce the value of M_{12}. Bringing them closer to the plane is another. Generally, any change that reduces the self inductance of either loop is likely to reduce the mutual inductance between them.

Radiated Coupling

Radiated coupling results when electromagnetic energy is emitted from a source, propagates to the far-field, and induces voltages and currents in another circuit. Unlike common impedance coupling, no conducted path is required. Unlike electric and magnetic field coupling, the victim circuit is not in the electromagnetic near-field of the source. Radiated coupling is usually the only significant coupling mechanism when the source and victim circuits (including all connected conductors) are separated by many wavelengths.

Of the four possible coupling mechanisms, radiated coupling is the one that seems to get the most attention. The idea that currents flowing in one circuit can induce currents in another circuit that is across the room or even miles away is fascinating to most of us. Maxwell's treatise on electromagnetism postulated the existence of electromagnetic waves back in 1864. He was able to calculate the velocity of propagation of these waves and describe wave reflection and diffraction. However, it was 25 years later before anybody was able to verify the existence of electromagnetic waves. Practical transmitters and receivers were not developed until the beginning of the 20th century. People viewed electromagnetic radiation as something nearly magical. The theory was difficult to comprehend, and the equipment required to transmit and receive signals was fairly complicated.

Today, we take wireless communication for granted. It is no longer viewed as magical, but the theory is still complex, and the equipment used to send and receive signals is still pretty sophisticated. This leads many engineers to believe that electromagnetic radiation is difficult to create and difficult to detect. However, virtually all circuits radiate, and most pick up detectable amounts of ambient electromagnetic fields. It is not necessary to attach an antenna to a circuit to make it radiate, the structure and location of most high frequency circuits allows them to function as their own antenna or to couple to nearby objects that function as efficient antennas.

The more difficult challenge for the designers of most electronic products is to design circuits that don't produce too much electromagnetic radiation. To understand how and why circuits exhibit unintentional electromagnetic emissions, it is helpful to review a few general concepts related to electromagnetic radiation and antenna theory.

Fields Produced by a Time-varying Current

Consider the short current filament illustrated in Figure 3.10. A current with amplitude, I, and angular frequency ω extends from $-\frac{\Delta z}{2} < z < \frac{\Delta z}{2}$. Of course, a real current could not start and stop abruptly like this; however realistic current distributions can be modeled by a superposition of current filaments like this one.

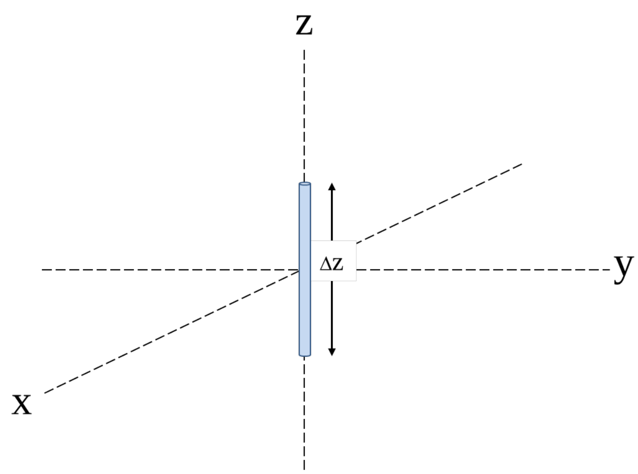


Figure 3.10. A small current filament.

The magnetic vector potential due to this current can be expressed as,

$$\vec{A} = \int_{-\frac{\Delta z}{2}}^{\frac{\Delta z}{2}} I \frac{e^{-j\beta r}}{r} dz \ \hat{z} \tag{3.41}$$

where the $e^{-j\beta r}$ term represents the delay between the time the current changes at the origin and the time the change can be detected at a point a distance r away. This equation mixes the spherical coordinate r with the Cartesian coordinate z. However, we will avoid this complexity by assuming that the length Δz is small relative to position r and the wavelength, λ. In this case, we can express the magnetic vector potential as,

$$\vec{A} \approx I \Delta z \frac{e^{-j\beta r}}{r} \hat{z} . \tag{3.42}$$

In free space, the magnetic field intensity can be calculated from the magnetic vector potential as,

$$\begin{aligned} \vec{H} &= \frac{1}{\mu_0} \nabla \times \vec{A} \\ &\approx \frac{I \Delta z \beta^2}{4\pi} e^{-j\beta r} \left[\frac{j}{\beta r} + \frac{1}{(\beta r)^2} \right] \sin\theta \ \hat{\phi} . \end{aligned} \tag{3.43}$$

Note that the magnetic field points in the $\hat{\phi}$ direction everywhere. It circulates around the axis of the current and has its maximum amplitude in the plane perpendicular to the axis of the current (sin θ = 1). There is no magnetic field on the *z* axis.

Applying Faraday's Law in point form, we can calculate the electric field as,

$$\begin{aligned}\vec{E} &= \frac{1}{j\omega\varepsilon_0}\nabla\times\vec{H}\\ &= \frac{I\Delta z\eta_0\beta^2}{4\pi}e^{-j\beta r}\left[\frac{j}{\beta r}+\frac{1}{(\beta r)^2}-\frac{j}{(\beta r)^3}\right]\sin\theta\ \hat{\theta}\\ &\quad -\frac{I\Delta z\eta_0\beta^2}{4\pi}e^{-j\beta r}\left[\frac{2}{(\beta r)^2}-\frac{2j}{(\beta r)^3}\right]\cos\theta\ \hat{r}.\end{aligned} \tag{3.44}$$

The electric field is perpendicular to the magnetic field at every point in space and has both a $\hat{\theta}$ component and an $\hat{r}$ component. Although these expressions are fairly complicated, we can appreciate the more important aspects of these field distributions by considering two separate cases: $\beta r \ll 1$ and $\beta r \gg 1$. The phase constant β is inversely proportional to the wavelength, $\beta = 2\pi/\lambda$. Therefore, the quantity βr is a measure of how far from the source we are relative to a wavelength,

$$\beta r = 2\pi\frac{r}{\lambda}. \tag{3.45}$$

If we are close to the source relative to a wavelength, then $\beta r \ll 1$ and the field terms with $(\beta r)^3$ in the denominator dominate. This region is referred to as the *near field* of the source. The near field of the current filament is dominated by the electric field.

When we are far from the source, $\beta r \gg 1$, the terms with (βr) in the denominator dominate. This is referred to as the *far field* of the source. If we discard all but the (βr) terms, we get the following expressions for the electric and magnetic far fields,

$$\vec{E} \approx j\frac{I\Delta z\eta_0\beta}{4\pi r}e^{-j\beta r}\sin\theta\ \hat{\theta} \tag{3.46}$$

and

$$\vec{H} \approx j\frac{I\Delta z\beta}{4\pi r}e^{-j\beta r}\sin\theta\ \hat{\phi}. \tag{3.47}$$

Note that in the far field, $\vec{E}$ and $\vec{H}$ are perpendicular to each other and perpendicular to the direction of propagation, $\hat{r}$. The fields are in phase with each other, and the ratio of their amplitudes is,

$$\frac{\left|\vec{E}_{\text{far field}}\right|}{\left|\vec{H}_{\text{far field}}\right|} = \eta_0 \tag{3.48}$$

at all points in space. These are the characteristics of an electromagnetic plane wave. Far from the source, where the spherical wave front is large relative to the size of the observer, the radiated field is essentially a uniform plane wave.

Quiz Question

If the radiated electric field strength 3 meters away from a small source is 40 dB(μV/m), what is the field strength 10 meters away from the same source in free space?

a.) 40 dB(μV/m)
b.) 30 dB(μV/m)
c.) 20 dB(μV/m)

To answer the question above, we note that, in the far field of a radiation source, the field strength is inversely proportional to the distance. Therefore, increasing the distance by a factor of 3.3 would reduce the field strength by a factor of 3.3. This is approximately a 10-dB reduction in the field strength, so the correct response would be 30 dB(μV/m).

Fields Produced by a Small Current Loop

Consider the small loop of current illustrated in Figure 3.11. This source can be modeled as four current filaments oriented to form a square loop. We'll let the current amplitude be I_0 and the angular frequency be ω as in the previous example. Using the principle of superposition, we can add the electric fields from each current filament to calculate the fields due to the loop. This is a straightforward (though somewhat tedious) process. For our purposes, the result is more interesting than the derivation, so only the results are presented here.

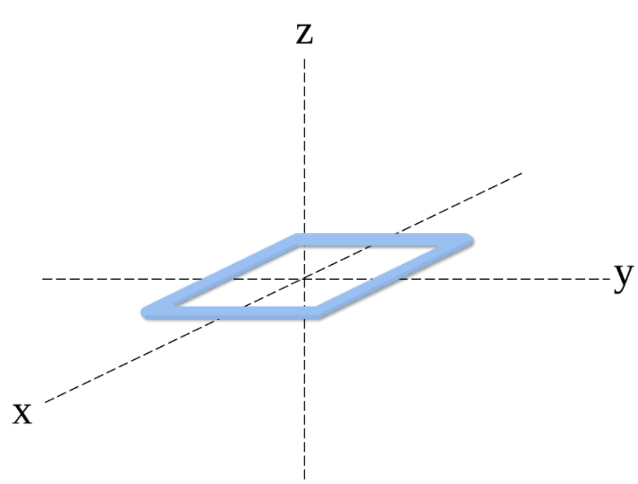


Figure 3.11. A small current loop.

In free space, the electric field intensity produced by a small loop of current is given by the expression,

$$\vec{E} = \frac{I\Delta s \eta_0 \beta^3}{4\pi} e^{-j\beta r} \left[\frac{-1}{\beta r} + \frac{j}{(\beta r)^2} \right] \sin\theta \ \hat{\phi} \tag{3.49}$$

where Δs is the area of the loop. Note the similarity between the electric field produced by the small loop and the magnetic field produced by the current filament (3.43). Both point in the $\hat{\phi}$ direction everywhere. Both are maximum at $\theta = 90^o$.

Applying Ampere's Law in point form, we can calculate the magnetic field as,

$$\begin{aligned} \vec{H} &= \frac{1}{j\omega\mu_0} \nabla \times \vec{E} \\ &= \frac{I\Delta s \beta^3}{4\pi} e^{-j\beta r} \left[\frac{-1}{\beta r} + \frac{j}{(\beta r)^2} - \frac{1}{(\beta r)^3} \right] \sin\theta \ \hat{\theta} \\ &\quad + \frac{I\Delta s \beta^3}{4\pi} e^{-j\beta r} \left[\frac{2j}{(\beta r)^2} + \frac{2}{(\beta r)^3} \right] \cos\theta \ \hat{r}. \end{aligned} \tag{3.50}$$

The magnetic field from the loop looks a lot like the electric field from the current filament (3.44). The magnetic field is perpendicular to the electric field at every point in space and has both a $\hat{\theta}$ component and a $\hat{r}$ component. If we are close to the source relative to a wavelength ($\beta r << 1$), the magnetic field dominates.

When we are far from the source ($\beta r >> 1$), we get the following expressions for the electric and magnetic far fields,

$$\vec{E} \approx \frac{I\Delta s \eta_0 \beta^2}{4\pi r} e^{-j\beta r} \sin\theta \ \hat{\phi} \tag{3.51}$$

and

$$\vec{H} \approx -\frac{I\Delta s \beta^2}{4\pi r} e^{-j\beta r} \sin\theta \ \hat{\theta}. \tag{3.52}$$

Again, we note that, in the far field, $\vec{E}$ and $\vec{H}$ are perpendicular to each other and perpendicular to the direction of propagation, $\hat{r}$. The fields are in phase with each other, and the ratio of their amplitudes is η_0.

Fields Produced by Electrically Small Circuits

Now let's apply what we know about the radiation from current filaments and current loops to estimate the radiation from an electrically small circuit. We'll start by considering the simple circuit illustrated in Figure 3.12. This circuit has an ideal voltage source and resistor connected by wire forming a loop with dimensions Δh and $\Delta \ell$. We'll assume that both Δh and $\Delta \ell$ are much less than the free-space wavelength, λ.

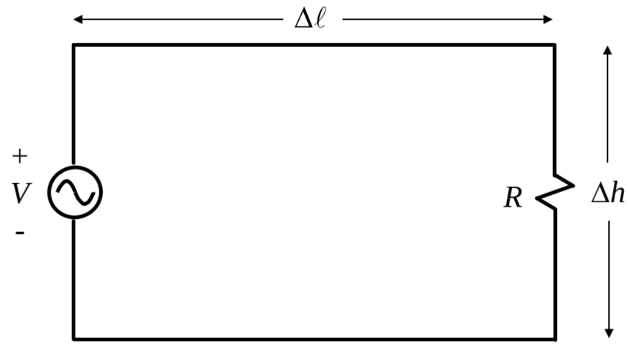


Figure 3.12. A simple circuit.

If the resistor has a very low value, we might expect this circuit to radiate much like a current loop. The current in the loop would be,

$$I = \frac{V}{Z_{LOOP}} = \frac{V}{R + j\omega L_{LOOP}} \tag{3.53}$$

where L_{LOOP} is the inductance of the rectangular loop. We can plug this expression for the current into Equation (3.51) to get an expression for the magnitude of the radiated electric field,

$$\begin{aligned} \left|\vec{E}\right| &\approx \left| j \frac{\Delta s \eta_0 \beta^2}{4\pi r} \frac{V}{Z_{LOOP}} e^{-j\beta r} \sin\theta \ \hat{\phi} \right| \\ &\approx \left| \frac{\Delta s \eta_0 \beta^2}{4\pi r} \frac{V}{Z_{LOOP}} \sin\theta \right|. \end{aligned} \tag{3.54}$$

Since we are normally interested in the maximum radiated field, regardless of orientation, we can replace the sin θ term with its maximum value of 1, resulting in,

$$\left|\vec{E}\right|_{\max} \approx \left| \frac{\Delta s \eta_0 \beta^2}{4\pi r} \left(\frac{V}{Z_{LOOP}} \right) \right|. \tag{3.55}$$

If R is a high impedance, the circuit does not look like a current loop. However, for very large values of R, we can model the circuit as three current filaments as illustrated in Figure 3.13. Radiation from the two horizontal current filaments connecting the source to the resistor is relatively low because the currents on these filaments are equal and opposite. However, there is a small amount of current flowing in the vertical section of the circuit on the source side. The amount of current flowing in the vertical part of the circuit can be estimated by viewing horizontal filaments as a short length of parallel-wire transmission line. Since the value of R is very large, the impedance at the source end of this transmission line is approximately equal to the input impedance of an open-circuited transmission line,

$$Z_{in} \approx Z_0 \cot \beta \ell \approx \frac{Z_0}{\beta \ell}. \tag{3.56}$$

Therefore, the current flowing in the vertical section on the left side of the circuit in Figure 3.13 is approximately,

$$I \approx \frac{V}{Z_{in}} \approx \frac{V \beta \ell}{Z_0}. \tag{3.57}$$

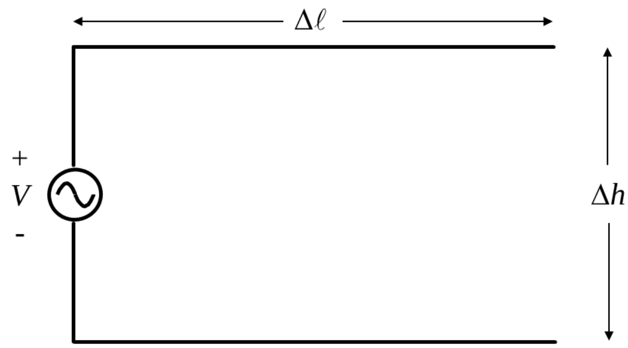


Figure 3.13. A simple high-impedance circuit.

If we substitute the above equation for I into the equation for calculating the far electric field from a current filament (3.46), we get,

$$\vec{E} \approx j \frac{V \Delta h \eta_0 \beta^2 \Delta \ell}{4 \pi r Z_0} e^{-j\beta r} \sin \theta \ \hat{\theta}. \tag{3.58}$$

We'll simplify this expression by noting that $\Delta h \times \Delta \ell = \Delta s$ and that the characteristic impedance of a parallel wire transmission line, Z_0, is typically a few hundred ohms, which is approximately the same as η_0. We'll also take the maximum value of the magnitude of this expression as we did for the low-impedance circuit, resulting in the following simple estimate of the maximum radiated field from a high impedance circuit,

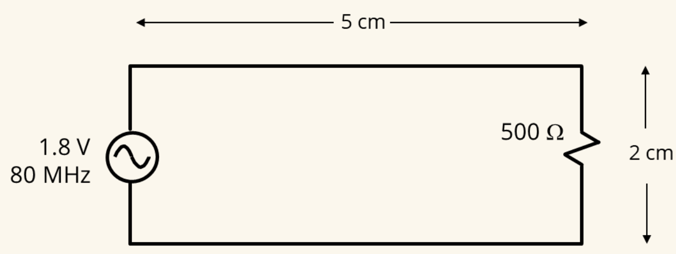
$$\left| \vec{E} \right|_{\max} \approx \frac{V \Delta s \beta^2}{4 \pi r}. \tag{3.59}$$

Note the similarity between the expression for high-impedance circuits (3.59) and the expression for low-impedance circuits (3.55). Both are proportional to the source voltage and the loop area. Both are proportional to the square of the frequency and inversely proportional to the distance from the source. The only difference between the two expressions is that the low-impedance circuit expression has an additional $\frac{\eta_0}{Z_{LOOP}}$ term. This suggests a practical method for distinguishing between a high-impedance circuit and a low-impedance circuit and we can estimate the maximum radiated electric field strength from any electrically small circuit using the following expression:

$$\left|\vec{E}\right|_{\max} \approx \begin{cases} \dfrac{V \Delta s \beta^2}{4\pi r} & Z_{LOOP} > \eta_0 \\ \dfrac{V \Delta s \beta^2}{4\pi r}\left(\dfrac{\eta_0}{Z_{LOOP}}\right) & Z_{LOOP} < \eta_0 \end{cases}. \tag{3.60}$$

Example 3-7: Estimating the radiated field from an electrically small circuit

Calculate the maximum radiated field from the circuit illustrated below. Do the emissions from this circuit exceed the FCC Class B limit?



wire radius: 0.5 mm

First, we need to determine whether the circuit is electrically small at the frequency of interest. At 80 MHz, the wavelength in free space is 3.75 meters. Since the maximum dimension of the circuit is much less than a wavelength, we can use (3.60) to estimate the maximum radiated field from this circuit.

The impedance is 500 Ω, which is greater than the 377-Ω intrinsic impedance of free space, so we use the upper equation in (3.60),

$$\begin{aligned} \left|\vec{E}\right|_{\max} &\approx \frac{V \Delta s \beta^2}{4\pi r} \\ &\approx \frac{(0.18\,V)(0.05 \times 0.02\,\text{m})\left(2\pi / 3.75\,\text{m}\right)^2}{4\pi(3\,\text{m})} \\ &\approx 13.4\,\mu\text{V/m} \quad \left[\text{or } 22.5 \text{ dB}(\mu\text{V/m})\right]. \end{aligned}$$

The FCC Class B limit at 80 MHz is 100 μV/m or 40 dB(μV/m), suggesting that this circuit would be 17.5 dB under the limit. However, the field calculated above is in free space and the FCC EMI testing is done in a semi-anechoic environment (above a ground plane). Reflections off the ground plane may add in-phase or out-of-phase with the radiation directly from the circuit. Since we are calculating the maximum radiation (and since the FCC scans the antenna height looking for the maximum), we should double our calculated field strength (i.e., add 6 dB) to account for the presence of the ground plane. In this case, our estimate of the maximum emissions from the circuit above a ground plane becomes 28.5 dB(μV/m) or 11.5 dB below the FCC Class B limit.

As the example above indicates, the presence of a ground plane complicates a radiation calculation. If the ground plane is infinite (or at least very large relative to a wavelength), the amplitude of the radiated field can be as much as twice its value without the ground plane.

How about the planes on a printed circuit board or the walls of a metal enclosure? Do they have the same effect? Generally, if the planes are much larger than a wavelength and much larger than the dimensions of the source, we can model the plane by placing an *image* of the source below the plane.

Figure 3.14 illustrates some simple current configurations and their images in a perfectly conducting plane. The image currents flowing perpendicular to the plane will be in the same direction as the source currents. The image currents flowing parallel to the plane are in the opposite direction of the source currents. This suggests that the fields from current sources parallel to and near the plane are decreased by the plane, while fields from current sources perpendicular to the plane are enhanced by the plane.

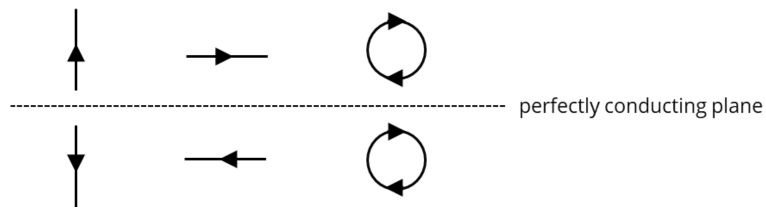


Figure 3.14. Current sources and their images in a conducting plane.

Efficient Sources of Electromagnetic Radiation

While it's convenient that we can readily calculate the radiated emissions from electrically-small circuits, these emissions tend to be weak. Reasonably well-designed circuits are very unlikely to produce radiated field strengths strong enough to exceed a radiated emissions specification. And yet, unintentional radiated emissions from electronics products can be a significant concern. If these emissions are not coming from the circuits, where are they coming from? What features of an electronic product make good unintentional antennas?

To help answer these questions, consider the current segment illustrated at the top of Figure 3.15. The segment has a length, Δz, and carries a uniform current, I_c. For now, we won't be concerned with the fact that currents can't suddenly appear and disappear. We can view this as one segment of a larger configuration.

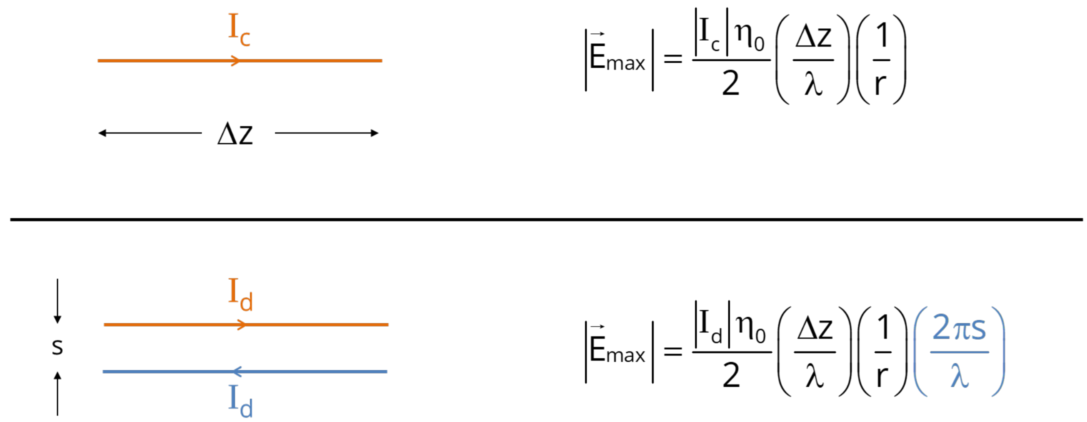


Figure 3.15. Maximum emissions from common-mode and differential-mode currents.

Starting with Equation (3.46) for the far electric field produced by a current filament, we can determine the maximum radiated field,

$$\begin{aligned} \left|\vec{E}_{\max}\right| &= \left| j\frac{I\Delta z\eta_0\beta}{4\pi r}e^{-j\beta r}\sin\theta \right| \\ &= \frac{|I_c|\Delta z\eta_0\beta}{4\pi r} \\ &= \frac{|I_c|\eta_0}{2}\left(\frac{\Delta z}{\lambda}\right)\left(\frac{1}{r}\right). \end{aligned} \tag{3.61}$$

Here we've chosen the field in the direction of maximum emissions ($\theta = 90^o$) and we've used the fact that $\beta = 2\pi/\lambda$ to express the result as a function of wavelength. The current was designated, I_c, recognizing that this could be one current on one wire or the common-mode current flowing on a wire bundle. Note that the maximum radiated field is proportional to the current, proportional to the length of the segment, and inversely proportional to the distance from the segment.

Now consider the case where there are two current segments separated by a distance, s, as illustrated in the lower half of Figure 3.15. The current on the lower segment has the same magnitude as the current on the upper segment, but it is flowing in the opposite direction. The current is designated, I_d, because it represents the differential-mode current flowing on a wire pair.

Starting with Equation (3.51) for the far electric field produced by a current loop, we can determine the maximum radiated field,

$$\left|\vec{E}_{\max}\right| = \left|\frac{I\Delta s\eta_0\beta^2}{4\pi r}e^{-j\beta r}\sin\theta\right|$$
$$= \frac{\left|I_d\right|\Delta s\eta_0\beta^2}{4\pi r} \tag{3.62}$$
$$= \frac{\left|I_d\right|\eta_0}{2}\left(\frac{\Delta z}{\lambda}\right)\left(\frac{1}{r}\right)\left(\frac{2\pi s}{\lambda}\right).$$

Here we've made the same substitutions as before, plus we've recognized that the loop area is, $\Delta s = \Delta z \times s$. At $\theta = 90^{o}$, the ends don't contribute to the radiated field, so the expression for the closed loop is the same as the expression for the two wire segments.

The terms in the final expressions in (3.61) and (3.62) have been rearranged to make them easy to compare. Note that the radiation from the differential-mode current is identical to the radiation from the common-mode current, except that the differential-mode expression has an extra term, $2\pi s/\lambda$. If the separation between the wire segments is much less than a wavelength, then the radiation due to a differential-mode current will be much less than the radiation due to the same amount of common-mode current. For any reasonably well laid out high-frequency signal current path, this will always be the case.

For example, for 30-MHz currents flowing on wire pair with a wire separation of 1 mm, the differential-mode emissions would be lower than the common-mode emissions by $2\pi s/\lambda = 2\pi \times 10^{-4}$ or 64 dB. In other words, to produce the same radiated field as 5 μA of common-mode current, the differential-mode current would have to be 8 mA. Twisting the wire pair would reduce the differential-mode emissions even more.

Signal currents in a transmission line are differential, and differential-mode currents rarely contribute significantly to radiated emissions problems. When a product fails to meet a radiated emissions requirement, it is almost always due to common-mode currents on a cable or structure. But common-mode currents can't suddenly appear on one end of a wire and disappear on the other end as they did in this example. The common-mode current must be zero at the ends of an unconnected wire. At a given frequency, the common-mode current distribution on a reasonably straight wire will be approximately sinusoidal with a wavelength equal to the free-space wavelength. If the current is zero at the end of a wire, it will reach its maximum value approximately a quarter-wavelength from the end.

This is illustrated in Figure 3.16 which shows the magnitude of the common-mode current on straight 6-meter wires driven by voltage sources located one meter from the left end. The height of the orange curve indicates the magnitude of the current. In each case, the current at the ends of the wire is zero. In each case, the current distribution varies sinusoidally moving away from the ends. In each case, the current must be continuous through the source. At 5 MHz, the wavelength is 60 meters, so the slowly varying sinusoidal current distributions on each side of the source are nearly straight lines. At 15 MHz, the wavelength is 20 meters, so the source is a quarter wavelength from the right end, but only 1/20th of a wavelength from the left end. At 75 MHz, the wavelength is 4 meters, so the source is a quarter wavelength from the left end and 5 quarter-wavelengths from the right end.

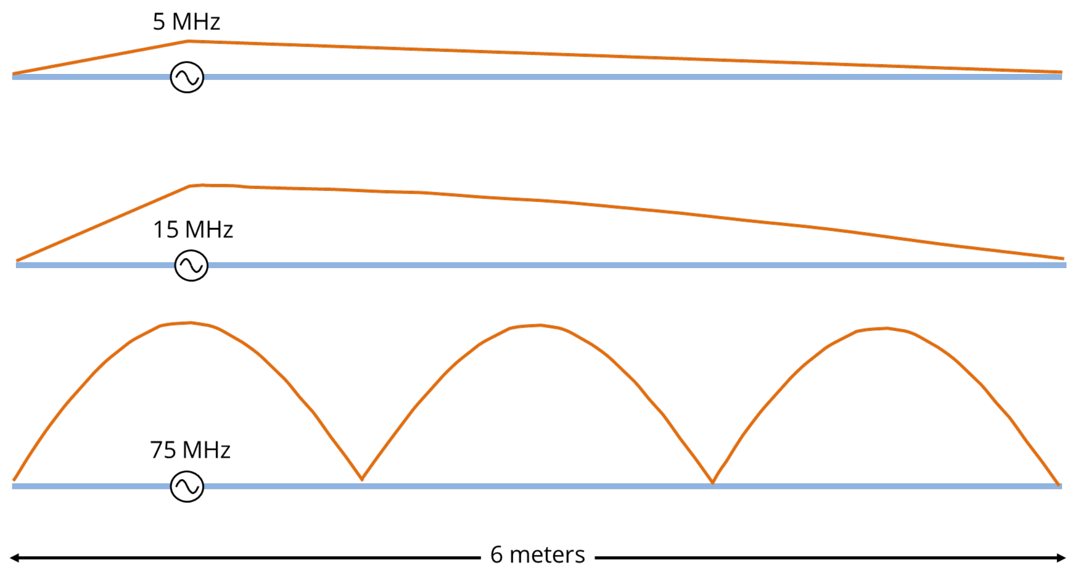


Figure 3.16. CM current distribution at three frequencies on a 6-meter wire.

In this example, the wire is resonant at 75 MHz and the source is located at a current maximum. In this location, it is relatively easy for the source to supply current. At lower frequencies, the source's ability to supply current is limited due to the electrically short 1-meter section of wire on the left side. If the 1-meter of wire on the left were replaced with a large metal structure (e.g., a large plate), the structure could supply the current necessary to drive the wire on the other side of the source. This *monopole* structure would resonate at frequencies where the wire on the right side was an odd multiple of a quarter wavelength.

Input Impedance and Radiation Resistance

In general, if a time-varying voltage difference appears between any two conducting objects in an open environment, time-varying currents will flow on these conductors and radiation will result. As indicated in the previous section, electrically small circuits are relatively inefficient sources of electromagnetic radiation. Larger resonant structures can produce radiated fields that are many orders of magnitude stronger when driven with the same voltage.

Consider the basic dipole antenna structure illustrated in Figure 3.17. A sinusoidal voltage source connected between two metal wires draws charge off one wire and pushes it onto the other wire when the voltage difference is positive. One half cycle later, the polarity is reversed, and the charge distribution is inverted. The moving charge results in a current. The ratio of the voltage to the current through the source is the *input impedance* of the antenna, which in general has a real and imaginary part,

$$Z_{in} = \frac{V_{in}}{I_{in}} = R_{in} + jX_{in}\,. \tag{3.63}$$

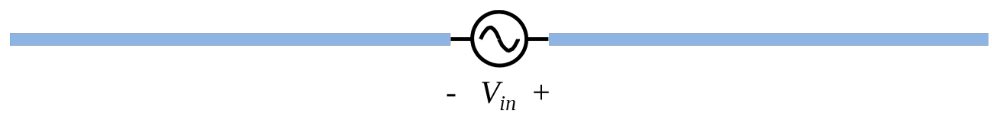


Figure 3.17. A simple dipole antenna geometry.

At low frequencies, the amount of charge the wires can hold for a given voltage difference is determined by the mutual capacitance between the wires. In this case, the imaginary part of the input impedance is,

$$X_{in} \approx \frac{1}{2\pi f C} \tag{3.64}$$

where f is the source frequency and C is the mutual capacitance. If the wires are good conductors, $R_{in} \approx 0$ at low frequencies and very little real power is delivered by the source.

However, as the frequency increases (and the wires look longer relative to a wavelength), several factors combine to change the antenna input impedance:

- The inductance associated with the currents flowing in the bars (and the associated magnetic field) begins to affect the reactive part of the input impedance,
- The resistive losses increase due to the skin effect,
- Power is lost to radiation which contributes to the real part of the input impedance.

It is convenient to express the real part of the input impedance as the sum of two terms,

$$R_{in} = R_{rad} + R_{diss} \tag{3.65}$$

where R_{rad} is the *radiation resistance* of the antenna and R_{diss} is the loss resistance. The power radiated can then be calculated as,

$$P_{rad} = \left|I_{in}\right|^2 R_{rad} \tag{3.66}$$

and the power dissipated as heat can be calculated as,

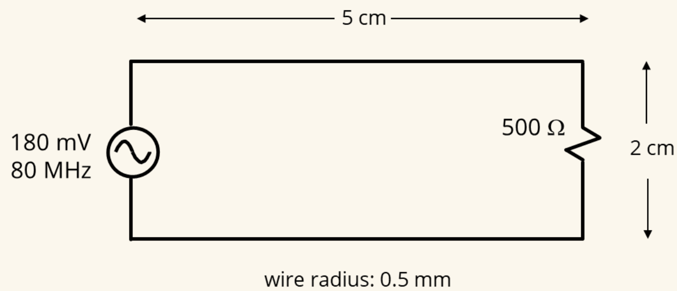
$$P_{diss} = \left|I_{in}\right|^2 R_{diss} \,. \tag{3.67}$$

The ratio of the power radiated to the total power delivered to the structure is called the radiation efficiency and can be calculated using the following equation,

$$e = \frac{P_{rad}}{P_{rad} + P_{diss}} = \frac{R_{rad}}{R_{rad} + R_{diss}} \,. \tag{3.68}$$

Example 3-8: Radiation Efficiency of an Electrically Small Circuit

Calculate the radiation efficiency of the 5-cm x 2-cm 500-Ω circuit illustrated below.



We'll start by calculating the power dissipated. If we assume that the power is primarily dissipated in the load resistor (as opposed to the wires), the dissipated power is simply,

$$P_{diss} = \left|I_{in}\right|^2 R_{diss} \approx \left|\frac{V_{in}}{R_{diss} + j\omega L}\right|^2 R_{diss} = \left|\frac{0.18}{500}\right|^2 500 = 65\ \mu\text{W}.$$

To estimate the power radiated, we note that the maximum electric field strength at 3 meters in free space is 13.4 μV/m (as calculated in Example 3-7). The maximum *radiated power density* is therefore,

$$\mathscr{P}_{rad} = \frac{\left|E\right|^2}{\eta_0} = \frac{\left|13.4 \times 10^{-6}\right|^2}{377} = 0.48\ \text{pW/m}^2.$$

This is the maximum power density radiated in any direction, so we can calculate an upper bound on the radiated power by assuming that this power density is radiated in all directions and integrating over a sphere with a 3-meter radius,

$$P_{rad} < \mathscr{P}_{rad}\left(4\pi r^2\right) = \left(0.48 \times 10^{-12}\right)\left(4\pi\right)\left(3\right)^2 = 54\ \text{pW}.$$

Therefore, the radiation efficiency of the circuit is,

$$e < \frac{54 \times 10^{-12}}{54 \times 10^{-12} + 65 \times 10^{-6}} = 8.3 \times 10^{-7} \text{ or } 0.000083\%.$$

Note that the input impedance of an antenna structure may depend on the antenna environment as well as the size and shape of the antenna. For example, the radiation resistance and the radiated power of any antenna will drop to zero if the antenna is operated in a fully-shielded resonant enclosure.

The Resonant Half-Wave Dipole

An antenna consisting of two simple conductors driven relative to each other by a single source is called a *dipole* antenna. A thin-wire antenna driven by a source at its center is called a *center-driven dipole*. The input impedance of a center-driven dipole is plotted in Figure 3.18 as a function of its electrical length (ℓ/λ).

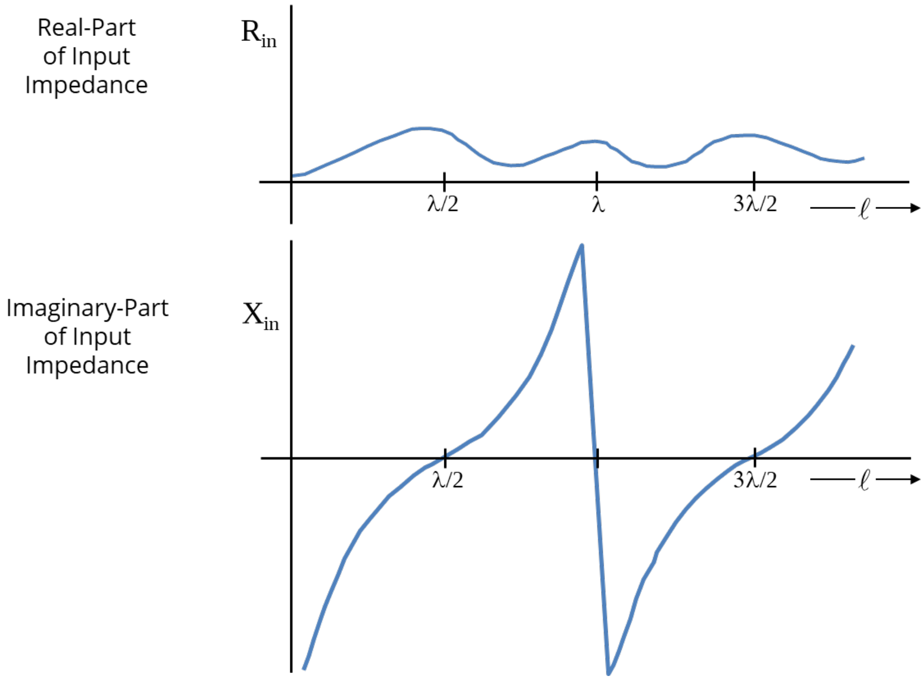


Figure 3.18. Input impedance of a center-driven dipole.

At very low frequencies (where $\ell << \lambda$), the input impedance is almost entirely reactive and inversely proportional to frequency ($Z_{in} \approx \frac{1}{2\pi f C}$). However, as the length (or frequency) increases, the magnitude of the negative reactance becomes smaller and eventually passes through zero before becoming positive and continuing to increase.

The reactance is zero when the total length of the wire is slightly less than one-half wavelength. A dipole antenna with this length has a real input impedance of approximately 72 Ω and is called a *half-wave resonant dipole*.

Quiz Question

Calculate the power radiated by a lossless half-wave resonant dipole driven by a 180-mV source.

This is a very simple calculation since both the input resistance and the radiation resistance are about 72 Ω. The correct solution is,

$$P_{rad} = \left|\frac{V}{R_{in}}\right|^2 R_{rad} = \left|\frac{0.18}{72}\right|^2 72 = 15\ \mu\text{W}. \tag{3.69}$$

To find the maximum radiated field strength at 3 meters from this antenna, we first determine the maximum radiated power density,

$$\wp_{rad_{\max}} = \frac{P_{rad}}{4\pi r^2} D_0 = \frac{15\times 10^{-6}}{4\pi(3)^2}(1.64) = 200 \text{ nW/m}^2. \quad (3.70)$$

where $\frac{P_{rad}}{4\pi r^2}$ is the average power density and $D_0 = 1.64$ is the directivity of a half-wave dipole antenna. The maximum radiated electric field can then be calculated using Equation (3.66) in reverse,

$$\left|\vec{E}_{rad_{\max}}\right| = \sqrt{\eta_0 \wp_{rad}} = \sqrt{(377)(200\times 10^{-9})} = 8.7 \text{ mV/m}. \quad (3.71)$$

Comparing this to the field strength radiated by the electrically small circuit in Example 3-7, we can gain an appreciation for just how important the size and shape of the antenna can be. In this case, if we assume both structures were driven at 80 MHz, the maximum dimension of the circuit was 5 cm while the maximum dimension of the dipole is 187.5 cm (half a wavelength at 80 MHz). This is a factor of 37.5. However, the radiated emissions increased by a factor of $\frac{8.7 \text{ mV/m}}{13.4\ \mu\text{V/m}} \approx 650$ or about 56 dB.

Example 3-9: Radiation Efficiency of a Half-Wave Dipole

Calculate the radiation efficiency of a center-driven half-wave resonant dipole made from copper wire with a radius (r=0.5 mm) at 100 MHz.

The power radiated by resonant half-wave dipole is simply,

$$P_{rad} = \frac{1}{2}|I|^2 (72\ \Omega) = 36|I|^2$$

where I is the peak current at the source. To calculate the power dissipated, we start by determining the resistance per unit length of the copper wire at 100 MHz.

$$R_{per\,unit\,length} = \frac{1}{(2\pi a)\delta\sigma} = \frac{\sqrt{\pi(100\times 10^6)(4\pi\times 10^{-7})(5.7\times 10^7)}}{2\pi(5\times 10^{-4})(5.7\times 10^7)} = 0.84\ \Omega/\text{m}.$$

The total power dissipated in the half-wave dipole is then,

$$P_{dissipated} = (0.84)\int_{-\frac{\lambda}{4}}^{+\frac{\lambda}{4}} |I \sin x|^2 dx = 0.42|I|^2.$$

Therefore, the efficiency of this resonant half-wave dipole is,

$$e = \frac{36|I|^2}{36|I|^2 + 0.42|I|^2} = 0.988 \text{ or } 98.8\%.$$

Compare this to the 0.000083% efficiency of the small circuit in Example 3-8. Resonant length wire antennas tend to be very efficient compared to electrically small antennas. They can easily be 4 to 6 orders of magnitude more efficient.

Quarter-Wave Monopoles

Half-wave dipoles make good antennas for many applications, but they are large at low frequencies and may not operate as intended close to a large metal surface. A *quarter-wave monopole* is simply half of a half-wave dipole driven relative to a large metal plane as illustrated in Figure 3.19. The lower half of the monopole can be modeled as the image of the upper half. Therefore, the radiating properties of the quarter-wave monopole are similar to those of the half-wave dipole. The input impedance of a resonant quarter-wave monopole is exactly half that of a resonant half-wave dipole or about 36 Ω.

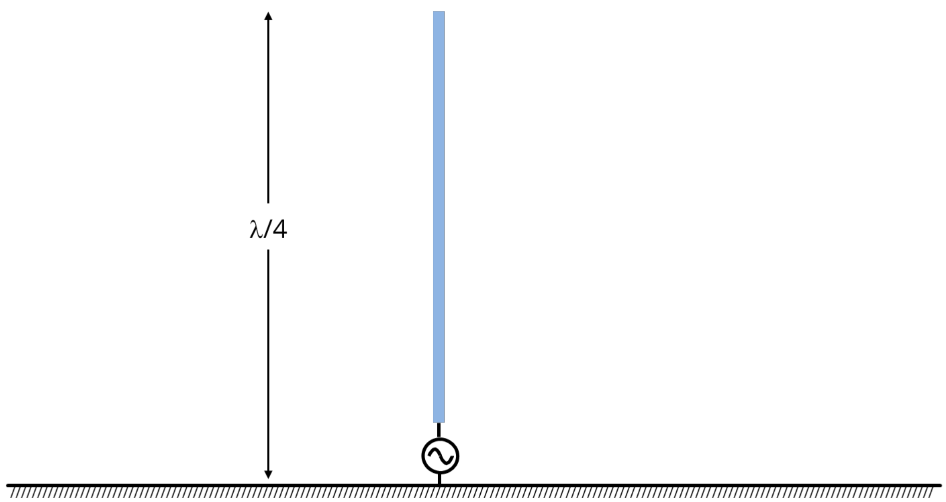


Figure 3.19. A quarter-wave monopole.

Cables driven relative to large metallic enclosures can often be modeled as monopole antennas. Since resonant monopole antennas are very efficient radiation sources, it is important to ensure that voltage differences between cables and enclosures are held to very small values at frequencies that might be near cable resonances.

Quiz Question

At approximately what frequency does a 25-cm wire attached to a large metal structure look like a quarter-wave monopole antenna?

The exact answer depends on the orientation of the wire, the cross section of the wire, the structure's size and shape, and other factors. However, 25 cm is a quarter wavelength at 300 MHz. The cable is likely to resonate and become an efficient antenna near this frequency.

Dipoles Driven Off-Center

When a wire antenna is driven off-center, it will still exhibit a resonance near the frequency where it is a half-wavelength long. However, the radiation resistance at resonance will be a function of the source location. Figure 3.20 illustrates how the radiation resistance of a resonant half-wave dipole changes as a function of source location. Note that the resistance quickly increases as the source is moved away from the center. A voltage source located near the end of the wire cannot deliver power effectively to the antenna even at resonance.

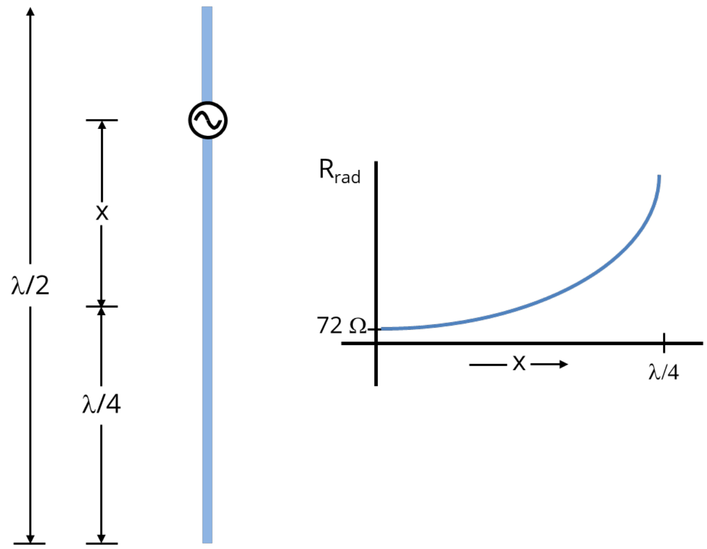


Figure 3.20. Radiation resistance of a resonant half-wave dipole as a function of source position.

Characteristics of Efficient and Inefficient Antennas

Most of the unintentional radiation sources that an EMC engineer encounters can be modeled as simple dipole antennas. There are basically 3 conditions that have to be met in order for these antennas to radiate effectively:

- The antenna must have two parts.
- Both parts must not be electrically small.
- Something must induce a voltage difference between the 2 parts.

The first condition is important to remember when trying to track down the source of a radiation problem. It is not correct to say that a particular wire or a particular piece of metal is “the antenna”. A single conductor will not be an effective antenna unless it is driven relative to something else. The “something else” is an equally important part of the antenna. Once both sides are identified, options for reducing radiated emissions normally become clearer.

Locating the two sides of an effective antenna becomes much easier when we recognize that they cannot be electrically small. For example, if we are looking for the “antenna” responsible for radiated emissions at 80 MHz (λ = 3.75 meters), then we are looking for 2 conducting objects on the order of a meter long. It is unlikely that these antenna parts are located on the printed circuit board. Most table-top size products can only radiate effectively at low frequencies by driving one cable relative to a chassis or another cable. At frequencies below a few hundred MHz, the number of possible antenna parts is very limited and readily apparent without a detailed examination of the entire design.

The third condition above suggests a method for controlling radiated emissions. Once the possible antenna parts have been identified, a device will not generate significant radiated emissions if the voltage difference between these parts is kept low. This is best accomplished by locating these parts near each other and ensuring that no high-frequency circuits come between them. Tying them together electrically with a good high-frequency connection will further ensure that they are held to the same potential.

Slot Antennas

Slot antennas are another potentially efficient type of antenna that EMC engineers should be familiar with. As illustrated in Figure 3.21, a slot antenna is formed by a long thin aperture in a conducting surface. An electric field distribution appearing in the slot (e.g., due to a surface current that is disrupted by the slot) produces a radiated field in the same way that a current distribution on a wire does. In fact, slot antennas are generally analyzed by replacing the electric field distribution with an equivalent (but fictitious) magnetic current and solving for the fields radiated by these magnetic currents. The fields radiated by a resonant half-wave slot have the same form (with the role of $\vec{E}$ and $\vec{H}$ reversed) as the fields from a resonant half-wave dipole. Like wire antennas, electrically small slots are inefficient antennas, while slots approaching a half-wavelength can be very efficient.

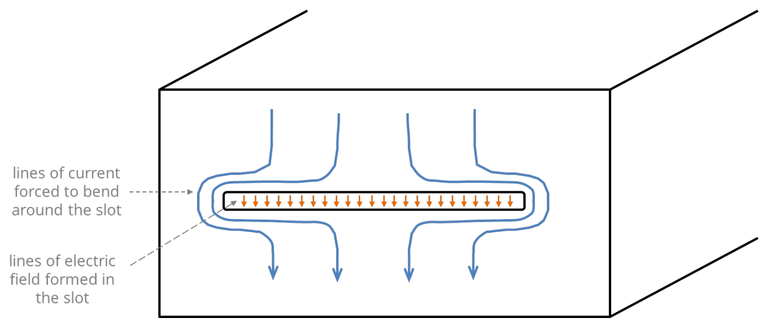


Figure 3.21. Slot antenna.

It's important to note that narrow apertures in the walls of a metal enclosure form a cavity-backed slot antenna. This can be a particularly efficient source of high-frequency radiated emissions with a resonant frequency that is a function of both the slot length and the cavity dimensions.

Receiving Antennas

Generally, the same structures that make good radiating antennas also make good receiving antennas. For this reason, many of the same techniques used to identify or prevent radiated emission problems can be applied to radiated susceptibility problems. However, unlike radiation, where the source impedance is almost always low relative to the antenna input impedance, the devices that exhibit susceptibility problems often have high impedance inputs. Because of this, it is not necessarily true that higher antenna input impedances correspond to poorer antenna performance.

The power received by a device connected to a dipole antenna can be calculated using the following formula,

$$P_{rec} = \wp_{rec} A_e \left(1 - |\Gamma|^2\right). \tag{3.72}$$

where,

$\wp_{rec} = \dfrac{\left|\vec{E}_{rec}\right|^2}{\eta}$ is the power density of the incident wave,

$A_e = \dfrac{\lambda^2}{4\pi} D_0$ is the effective aperture of the antenna,

$\left(1 - |\Gamma|^2\right)$ is the factor accounting for the impedance mismatch between the antenna and the receiver, and

$\Gamma = \dfrac{Z_{receiver} - Z_{antenna}}{Z_{receiver} + Z_{antenna}}$ is the voltage reflection coefficient at the receiver.

This formula may be difficult to apply in many situations because it requires significant information about both the antenna and the receiver. If an order-of-magnitude approximation is good enough, it is convenient to estimate the maximum voltage dropped across a high impedance input as,

$$V_{rec} \approx \left|\vec{E}_{inc}\right| \ell_{ant}. \tag{3.73}$$

where ℓ_{ant} is the length of the dipole antenna or ½ wavelength, whichever is greater.

Example 3-10: Estimating the Maximum Voltage Coupled to a Half-Wave Dipole

Compare the actual maximum voltage coupled to a 500-Ω receiver from a half-wave dipole to the estimate in Equation (3.73).

If we assume that the receiver is positioned at the point on the dipole where its impedance is matched to the radiation resistance, the expression for the received power becomes,

$$P_{rec} = \frac{\left|\vec{E}_{inc}\right|^2}{\eta_0}\left(\frac{\lambda^2}{4\pi} D_0\right) = \frac{\left|\vec{E}_{inc}\right|^2}{377}\left(\frac{\lambda^2}{4\pi}\right)(1.64) = 3.5\times10^{-4}\lambda^2\left|\vec{E}_{inc}\right|^2 \text{ W.}$$

The received voltage is,

$$V_{rec} = \sqrt{R_{in} P_{rec}} = \sqrt{(500)\left(3.5\times10^{-4}\right)}\,\lambda\left|\vec{E}_{inc}\right| = 0.4\lambda\left|\vec{E}_{inc}\right|.$$

We can compare this to the estimated value,

$$V_{rec} = 0.5\lambda\left|\vec{E}_{inc}\right|$$

and note that the estimate was accurate to within 2 dB in this case.

Example 3-11: Estimating the Maximum Voltage Coupled to a Short Dipole Antenna

Compare the actual maximum voltage coupled to a matched receiver from an electrically short dipole to the estimate in Equation (3.73).

The radiation resistance of an electrically short dipole antenna is approximately,

$$R_{rad} \approx 20\pi^2\left(\frac{\ell}{\lambda}\right)^2.$$

The directivity, D_0, is 1.5. The received power can be readily calculated as,

$$P_{rec} = \frac{\left|\vec{E}_{inc}\right|^2}{\eta_0}\left(\frac{\lambda^2}{4\pi}D_0\right) = \frac{\left|\vec{E}_{inc}\right|^2}{377}\left(\frac{\lambda^2}{4\pi}\right)(1.5) = 3.2\times10^{-4}\lambda^2\left|\vec{E}_{inc}\right|^2 \text{ W}.$$

The received voltage is therefore,

$$V_{rec} = \sqrt{R_{in}P_{rec}} = \sqrt{\left(20\pi^2\right)\left(\frac{\ell}{\lambda}\right)^2\left(3.2\times10^{-4}\right)}\lambda\left|\vec{E}_{inc}\right| \approx 0.25\ell\left|\vec{E}_{inc}\right|.$$

We can compare this to the estimated value,

$$V_{rec} = \ell\left|\vec{E}_{inc}\right|.$$

In this case, the voltage is over-estimated by a factor of 4 (or 12 dB).