EMC Course Notes 2024

Parasitic Impedance Parameters

Resistance

In a circuit schematic, components are connected by lines. During a circuit simulation, those lines are assumed to be equipotential. No matter how much current they carry, the voltage on the line is the same everywhere.

In real, physical circuits, the lines on a schematic diagram are implemented using wires, traces on a circuit board, or metallization in a component. The circuit connections are made using materials such as gold, copper or aluminum. These materials are good conductors, but they are not perfect conductors. The finite conductivity of circuit connections ensures that they will have a non-zero resistance.

Most of the time, this resistance has a negligible effect on the circuit operation. However, there are many situations where the resistance of circuit connections can be important. When reviewing a circuit or system design, EMC engineers must be able to quickly estimate the resistance of various circuit interconnects and determine whether this resistance is likely to be a factor affecting possible coupling or noise problems.

Fortunately, calculating the resistance of a circuit interconnect is not difficult, particularly if we only require order-of-magnitude accuracy.

Quiz Question

The D.C. resistance of a 5-cm trace on a printed circuit board is,

a.) about 100 Ω
b.) about 100 mΩ
c.) less than 1 mΩ

In circuit theory, Ohm's law relates the voltage difference across a resistance to the current passing through it,

$$V = IR \text{ volts.} \tag{2.1}$$

There is also a point form of Ohm's law that is an important part of electromagnetic field theory. In point form, Ohm's law relates a current density to the electric field at a point,

$$\vec{J} = \sigma \vec{E} \quad \text{A/m}^2 \tag{2.2}$$

where σ is the conductivity (in siemens/m) of the material in the region of the point.

The resistance of a cylindrical object with a uniform current density such as that illustrated in Figure 2.1 can be found by integrating the point form of Ohm's law over a cross section,

$$\int_S \vec{J} \cdot \vec{ds} = \int_S \sigma \vec{E} \cdot \vec{ds} \quad \text{amperes.} \tag{2.3}$$

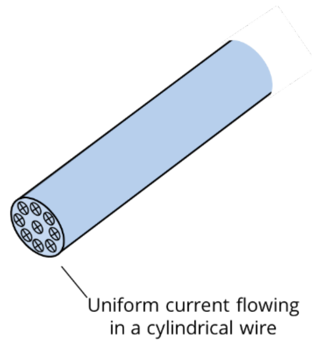


Figure 2.1. Cylindrical Wire with Uniform Current Density.

The left-hand side of the above equation is the total current flowing through the cross section. If the electric field strength is constant, then its magnitude can be expressed as the voltage from one end of the object to the other divided by the length, $\left|\vec{E}\right| = V / \ell$, and Equation (2.3) can be written as,

$$I = \frac{V}{\ell}\int_S \sigma \, ds \text{ amperes.} \tag{2.4}$$

Or, by rearranging,

$$\frac{V}{I} = \frac{\ell}{\int_S \sigma \, ds} \text{ ohms.} \tag{2.5}$$

If the conductivity, σ, is a constant, then the resistance can be expressed as a function of the cross-sectional area, $A = \int_S ds$, as follows,

$$R = \frac{\ell}{\sigma A} \text{ ohms.} \tag{2.6}$$

Example 2-1: DC Resistance of a Printed Circuit Board Trace

Suppose we want to calculate the resistance of a printed circuit board trace that is 5 cm long. The trace is made of copper with a conductivity, $\sigma = 5.7\text{x}10^7$ S/m. If the trace width is 0.25 mm and the trace thickness is 0.034 mm, then the resistance can be calculated from Equation (2.6) as,

$$R = \frac{0.05 \text{ m}}{(5.7\times10^{7} \text{ S/m})(0.25\times10^{-3} \text{ m})(0.034\times10^{-3} \text{ m})} = 0.10\ \Omega$$

or about 100 mΩ. Looking at the equation for resistance, Equation (2.6), we can see that longer traces would have a higher resistance, and wider or thicker traces would have a lower resistance.

The derivation of Equation (2.6) assumes that the current is uniformly distributed throughout the cross section of the conductor. This will generally be true for low-frequency currents (e.g., kHz frequencies and lower); however, at high frequencies, electric fields (and therefore electric currents) have a difficult time penetrating conductive materials. In the absence of other nearby conductors, the current density at high frequencies will peak at the surface and fall off exponentially inside the conductor as illustrated in Figure 2.2. The rate at which the current density decays depends on the frequency of the current, the conductivity, σ, and the permeability, μ, of the conductive material. In a good conductor ($\sigma >> \omega\varepsilon$), the current density inside a thick conductor is given by,

$$J(x) = J_S e^{-x\sqrt{\pi f \mu\sigma}} \text{ A/m} \tag{2.7}$$

where J_s is the current density on the surface (at x=0) and x is the distance from the surface. The distance at which the current density decays to $1/e$ of its value on the surface is called the *skin depth*, and is given by,

$$\delta = \frac{1}{\sqrt{\pi f \mu\sigma}} \text{ meters.} \tag{2.8}$$

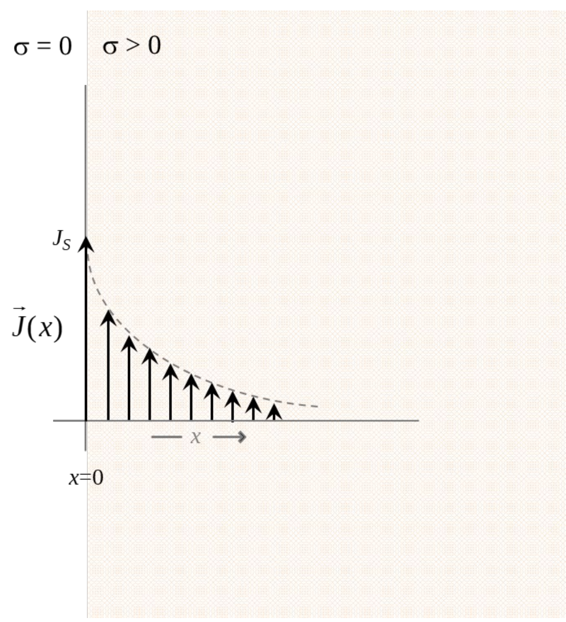


Figure 2.2. Current Density near the Surface of a Good Conductor.

The total current flowing in a conductor that is many skin depths thick is approximately,

$$I = \int_0^\infty J_S e^{-x/\delta} dx = J_S \delta \text{ amperes.} \tag{2.9}$$

This is the same as the total current that would flow if the current were constant, but only penetrated the surface a distance of one skin depth. When conductors are many skin depths thick, simple and accurate calculations of resistance can be obtained by making this approximation. For example, to calculate the resistance of a round wire at high frequencies, we model the current distribution as having a constant amplitude within one skin depth of the surface and zero amplitude everywhere else, as illustrated in Figure 2.3. Using this approximation, a round wire of radius a has a total wire current of,

$$J_S\left[\left(\pi a^2\right)-\pi\left(a-\delta\right)^2\right]\approx J_S\,2\pi a\,\delta \text{ amperes.} \tag{2.10}$$

The resistance of a round wire with length ℓ and radius a is then,

$$R\approx\frac{\ell}{\sigma 2\pi a\delta}\text{ ohms.} \tag{2.11}$$

Notice that Equation (2.11) is similar to the expression for the resistance at DC in Equation (2.6), except that the cross-sectional area of the wire, A, has been replaced by the current-carrying cross-sectional area, $2\pi a\delta$.

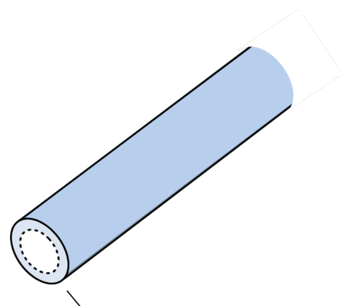
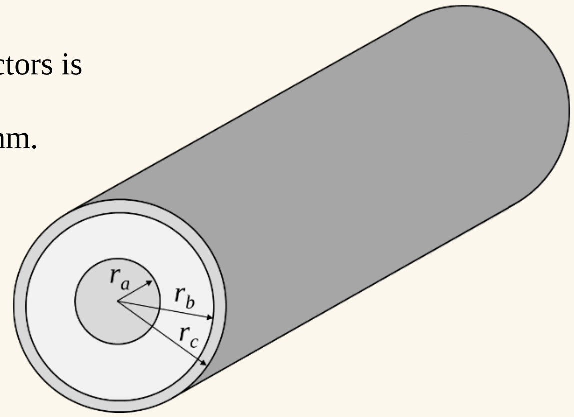


Figure 2.3. High-frequency current distribution in a round wire.

There are several other factors besides skin effect that can cause the current distribution in a conductor to be non-uniform. If the current distribution is known or can be approximated, then Equation (2.3) can be used to find the resistance of the conductor. If the current distribution is not known, numerical techniques can be employed. However, even when the precise current distribution is not known, an estimate of the resistance based on the equations above is often sufficient.

Example 2-2: Resistance per Unit Length of a Coaxial Cable

Calculate the resistance per unit length of the coaxial cable illustrated below at 60 Hz and at 100 MHz. Here, we'll assume the metal is copper with $\sigma = 5.7x10^7$ *S/m,* $\mu = \mu_0 = 4\pi x10^{-7}$ *H/m, and that* $r_a = 0.5$ *mm,* $r_b = 4.9$ *mm, and* $r_c = 5.0$ *mm.*



At 60 Hz, the skin depth of the copper conductors is

$$\delta_{60Hz} = \frac{1}{\sqrt{\pi(60)(4\pi \times 10^{-7})(5.7 \times 10^{7})}} = 8.6 \text{ mm.}$$

Since the thickness of both the inner and outer conductor in this coaxial cable is much less than this calculated skin depth, we will approximate the current density as being uniformly distributed within the conductors. The resistance per unit length of the cable is found from Equation (2.6) to be,

$$R_{InnerConductor60Hz} = \frac{1}{\sigma A} = \frac{1}{(5.7 \times 10^{7})\pi(0.5 \times 10^{-3})^2} = 22.3 \text{ m}\Omega/\text{m}$$

$$R_{OuterConductor60Hz} = \frac{1}{\sigma A} = \frac{1}{(5.7 \times 10^{7})2\pi(5 \times 10^{-3})(0.1 \times 10^{-3})} = 5.6 \text{ m}\Omega/\text{m.}$$

Since the current on a coaxial cable will typically travel out on one of the conductors and return on the other, these two resistances are encountered in series. Thus, the total resistance per unit length at 60 Hz is the sum of the resistance of the inner and outer conductors,

$$R_{60Hz} = 22.3 + 5.6 \simeq 28 \ \text{ m}\Omega/\text{m} .$$

At 100 MHz, the skin depth of copper is

$$\delta_{100MHz} = \frac{1}{\sqrt{\pi(10^{8})(4\pi \times 10^{-7})(5.7 \times 10^{7})}} = 0.0067 \text{ mm.}$$

Since the skin depth at 100 MHz is much less than the thickness of the conductors of the coaxial cable, we will use the approximation of Equation (2.11) to calculate the resistance per unit length for the inner and outer conductors,

$$R_{InnerConductor100MHz} = \frac{1}{\sigma 2\pi a\delta} = \frac{1}{\left(5.7\times10^{7}\right)2\pi\left(0.5\times10^{-3}\right)\left(6.7\times10^{-6}\right)} = 838 \text{ m}\Omega/\text{m}$$

$$R_{OuterConductor100MHz} = \frac{1}{\sigma 2\pi a\delta} = \frac{1}{\left(5.7\times10^{7}\right)2\pi\left(4.9\times10^{-3}\right)\left(6.7\times10^{-6}\right)} = 85 \text{ m}\Omega/\text{m}$$

$$R_{100\,MHz} = 838 + 85 = 923 \text{ m}\Omega/\text{m.}$$

Thus, for the coaxial cable in this example, signals at 60 Hz see a resistance per unit length of 28 mΩ/m, while signals at 100 MHz see a larger resistance per unit length of 923 mΩ/m. This example illustrates that the resistance of a coaxial cable depends on the frequency of the signals appearing on the cable, with higher frequency signals seeing a higher resistance.

Capacitance

Although capacitance is essentially a static field concept, it is fundamental to the description and analysis of both static and time varying configurations. It's not always important to be able to calculate capacitances precisely, but understanding what capacitance is, and being able to estimate it, are important skills for any EMC engineer.

Capacitance Between Two Conductors

In a system consisting of two conducting objects, applying a voltage (V) between the two objects would pull a charge (Q) from the object at the lower potential and put it on the object with the higher potential. The amount of charge would be directly proportional to the applied voltage, so the ratio of the two quantities would be a constant. This constant is referred to as the *mutual capacitance* between the two conductors and is given by,

$$C = \frac{Q}{V} \text{ farads} \tag{2.12}$$

where V is the electrostatic potential between the two conductors and Q is the magnitude of the equal and opposite charge on the two conductors.

The mutual capacitance is a function of the conductor geometry and the permittivity of the dielectric between them. Lines of electric field start on the conductor with the higher potential and terminate on the conductor with the lower potential. For a given voltage, the mutual capacitance quantifies the total electric flux between the two conductors and plays a vital role in quantifying electric-field coupling.

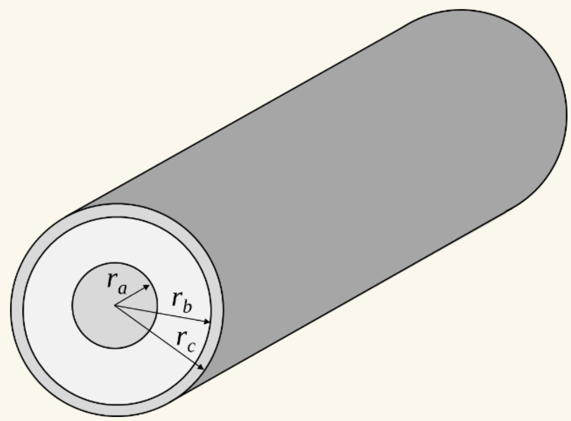
If the voltage increases in amplitude, the energy stored in the electric flux must also increase. This added energy must come from the source, so the capacitance of a pair of conductors represents an impedance to an increasing voltage. Likewise, if the voltage decreases, energy stored in the electric flux must be absorbed back into the source which tends to prevent the voltage from decreasing. Therefore, capacitance is a property that tends to resist changes in the amplitude of the voltage.

For simple geometries with a great deal of symmetry, we can calculate the capacitance between the two conductors by placing an opposing charge on them, determining the electric field from the charge distribution, and calculating the voltage by integrating the electric field over a path from one conductor to the other. This works well for large parallel plates, concentric spheres, and concentric cylinders (as illustrated in Example 2-3). For most other geometries, the mutual capacitance must be estimated by comparing to configurations with a known capacitance or determined using numerical modeling techniques.

Example 2-3: Capacitance per Unit Length of a Coaxial Cable

Find the capacitance per unit length of the coaxial cable shown below.

We will assume that the charge density is uniform, so that we can assume a line charge density of ρ_l coulombs per meter on the inner conductor. Applying Gauss' Law and taking advantage of the symmetry, the electric field between the cylinders is found to be,



$$\vec{E} = \frac{\rho_l}{2\pi\varepsilon r}\hat{r} \quad r_a < r < r_b \quad \text{V/m.}$$

Integrating the electric field from the inner cylinder to the outer cylinder, the potential between the two conductors is,

$$V_{ab} = \int_{r_a}^{r_b} \frac{\rho_l}{2\pi\varepsilon r}\,dr = \frac{\rho_l}{2\pi\varepsilon}\ln\left(\frac{r_b}{r_a}\right) \quad \text{volts.}$$

The capacitance per unit length is the charge per unit length divided by the potential,

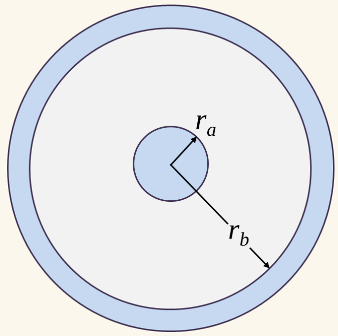
$$C_{ab} = \frac{\rho_l}{V_{ab}} = \frac{2\pi\varepsilon}{\ln\left(\frac{r_b}{r_a}\right)} \quad \text{F/m.}$$

Note that the geometry in Example 2-3 is that of a coaxial transmission line, so the result shown is the expression for the capacitance per unit length of a coaxial cable. Generally, the mutual capacitance between two objects will increase as the distance between the two objects decreases. It will also be proportional to the permittivity of the dielectric, with the smallest possible permittivity being that of free space, $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m.

Capacitance of One Conductor

While mutual capacitance is an important concept for evaluating conductors in very close proximity, there is another capacitance that plays an equally important role in EMC engineering. To illustrate the concept of *absolute capacitance*, we will start by calculating the mutual capacitance between two concentric spheres.

Example 2-4: Concentric Conducting Spheres



Consider the two conductive spheres illustrated above. If the charge on the inner sphere is Q_0 coulombs then the electric field between the spheres can easily be determined by applying Gauss' Law and taking advantage of the structure symmetry,

$$\vec{E} = \frac{Q_0}{4\pi\varepsilon_0 r^2}\hat{r}, \quad r_a < r < r_b \quad \text{V/m.}$$

The voltage between the spheres is found by integrating the electric field,

$$V_{ab} = \int_{r_a}^{r_b} \frac{Q_0}{4\pi\varepsilon_0 r^2}\,dr = \frac{Q_0}{4\pi\varepsilon_0}\left[\frac{1}{r_a} - \frac{1}{r_b}\right] \quad \text{volts}$$

and the capacitance is then shown to be,

$$C_{ab} = \frac{Q_0}{V_{ab}} = \frac{4\pi\varepsilon_0}{\left[\frac{1}{r_a} - \frac{1}{r_b}\right]} \quad \text{farads.}$$

Note that as the radius of the outer sphere approaches infinity, the capacitance of the system does not approach zero. An absolute capacitance can be defined for the inner sphere as,

$$C_{abs} = \lim_{r_b \to \infty} C_{ab} = 4\pi\varepsilon_0 r_a \quad \text{farads.}$$

The result above indicates that a sphere (or a finite object of any shape) holds a given amount of charge whenever its voltage (relative to infinity) is non-zero. The amount of charge is proportional to the voltage, but independent of the location of other distant conductors.

Absolute capacitance in general can be defined as the ratio of the charge on an object relative to its absolute potential. It is a measure of the total charge on an object at a given potential when the conductor is remote from other conductors and charges.

This is an important concept for EMC engineers, because it illustrates how any conducting object can become charged even when it is not near another conducting object. For example, a person dragging their rubber soled shoes across a wool carpet on a dry day can become charged to ~25,000 volts. The amount of charge they are carrying depends on their absolute capacitance, $Q_{stored} = C_{abs}$ x 25,000 volts.

Quiz Question

How much charge does it take to raise a person's potential to 25,000 volts?

a.) about 1 coulomb
b.) a few microcoulombs
c.) a few picocoulombs

A person curled up in a ball with a radius of about 0.5 meters will have an absolute capacitance of about $C_{abs} = 4\pi\varepsilon_0 r_a \approx 60$ pF. If the person stands erect, their absolute capacitance will be higher (e.g., 100 pF), because the charge can spread out more. A 100-pF person charged to 25,000 volts will hold approximately $Q = (100 \times 10^{-12}$ F) (25,000 V) = 2.5 μC of charge.

Self and Mutual Capacitance

It can be helpful to view absolute capacitance as a capacitance to infinity. Lines of electric field originating on the object terminate infinitely far away. In the real world, if we put charge on an object, it has always been pulled from somewhere else. So, let's consider the case where charge is pulled from one sphere and put on another nearby sphere as illustrated in Figure 2.4. There is a positive charge, Q, on one sphere and an equal amount of negative charge on the other sphere. The ratio of the charge to the voltage between the spheres is the mutual capacitance between the spheres.

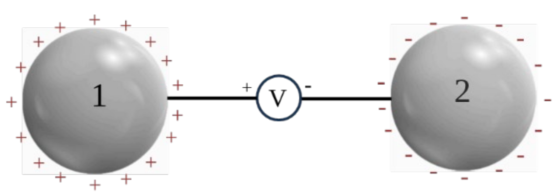


Figure 2.4. Two spheres with an applied voltage between them.

When the spheres are very close, the capacitance between the spheres is relatively high. This capacitance decreases as the spheres are moved farther apart, but there is a limit to how small the capacitance can become. Each sphere has a capacitance to infinity, so no matter where the spheres are located, it will always hold a charge proportional to its absolute voltage.

Schematically, we can model this system with three capacitances as illustrated in Figure 2.5. In the model, the voltage at infinity is zero, so lines of electric flux that terminate at infinity are modeled as a self capacitance to the zero-volt reference (electrical ground). Lines of flux that originate on one sphere and terminate on the other sphere are represented by the partial mutual capacitance[1], C_{12}. If the capacitance between the two spheres were measured, we would expect the measured value to be,

$$C_{meas} = C_{12} + \left(\frac{C_1 C_2}{C_1 + C_2} \right). \tag{2.13}$$

The flux either flows through C_{12} or through the series combination of C_1 and C_2.

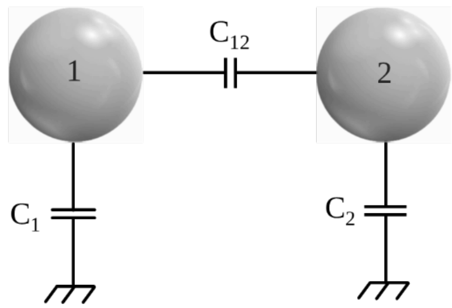


Figure 2.5. Schematic representation of self and partial mutual capacitances .

If the spheres are very close to each other relative to their diameter, most of the electric flux is concentrated in short lines that start on one sphere and terminate on the other sphere. The flux extending far from the immediate vicinity of the spheres is relatively weak. In this case, the value of C_{12} is relatively high while the values of C_1 and C_2 are small.

As the spheres are moved farther apart, the value of C_{12} decreases. Less of the flux emanating from one sphere is immediately captured by the other sphere. More of the flux extends far from the system where the relative position of the two spheres is unimportant, so the values of C_1 and C_2 increase. Once the distance between the two spheres is great enough (e.g., 5-10 sphere diameters), C_1 and C_2 are approximately equal to the absolute capacitances of their respective spheres. The value of C_{12} decreases to the point where it is much smaller than the series combination of C_1 and C_2. At this point, the distance between the two spheres no longer has a significant effect on their measured mutual capacitance.

Now let's examine what happens when the diameter of one sphere is greatly increased while maintaining an edge-to-edge separation larger than the smaller sphere's diameter. For example, as the sphere on the left in Figure 2.5 grows, the value of C_1 gets larger. On the other hand, the value of C_2 starts to fall while the value of C_{12} starts to increase. More lines of flux emanating from the smaller sphere are captured by the larger sphere and fewer of them extend to infinity.

In the limit as the first sphere grows infinitely large, C_1 becomes infinite, and the first sphere becomes the zero-volt reference. At the same time, C_2 goes to zero. All the flux from sphere 2 is captured by sphere 1. However, if the distance between sphere 1 and sphere 2 is much greater than the diameter of sphere 2, the total flux emanating from sphere 2 (for a given absolute voltage) is the same. The value of C_{12} is the absolute capacitance of C_2. In other words, when one conductor is much larger than the other, the mutual capacitance between them depends only on the size and location of the smaller conductor.

[1] Note that for two spheres in the middle of an empty universe, all the lines of flux originating on one sphere eventually terminate on the other. The term partial mutual capacitance is introduced to represent the part of the mutual capacitance that approaches zero as the distance between the spheres is increased.

And if the distance between the two conductors is large compared to the size of the smaller conductor, the mutual capacitance is essentially equal to the absolute capacitance of the smaller conductor.

For example, suppose the larger sphere is the earth (diameter ~12,800 km) and the smaller sphere is a soccer ball (diameter ~22 cm). Suppose the soccer ball is 1 meter above the ground. Strictly speaking, the soccer ball's capacitance to infinity is zero; however, its capacitance to the earth is equal to its absolute capacitance. In other words, the soccer ball holds the same amount of charge for a given voltage whether the earth is present or not.

In the literature, the term *capacitance to earth* is sometimes used to describe an absolute capacitance or a self capacitance. Technically, this is correct if it is describing a system anywhere within a few thousand km of the earth. Nevertheless, the term can be confusing because it implies that the presence of the earth has something to do with the value of this capacitance.

Capacitance in a System of Conductors

If we have several conductive objects, we can define a self capacitance value for each conductor and a partial mutual capacitance[2] between each conductor pair. Figure 2.6 illustrates this for a system of 3 conductors. To calculate these self and mutual capacitances, the principle of superposition is applied. The absolute potential of each conductor can be expressed as the sum of the potentials due to charge on each of the other conductors and its own charge. For n conductors,

$$V_i = \sum_{j=1}^{n} p_{ij} Q_j \quad \left(i = 1,2,...,n\right) \text{ volts} \tag{2.14}$$

where the coefficients of potential, p_{ij}, are functions of the geometry. This equation can be written in matrix form as,

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_n \end{bmatrix}. \tag{2.15}$$

[2] The term partial mutual capacitance was introduced to distinguish it from the term mutual capacitance as defined for a two-conductor system. However, in systems of conductors, the word partial is commonly dropped. Through the rest of this chapter the term mutual capacitance will refer to the partial mutual capacitance in a system of conductors. The term measured capacitance will be used to describe the capacitance between two conductors due to all available flux paths.

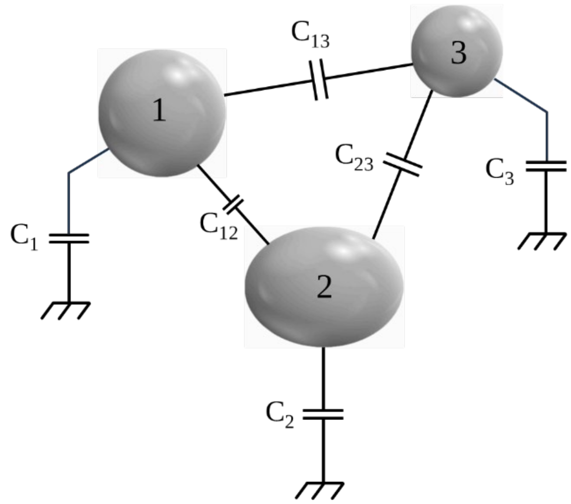


Figure 2.6. Self and Mutual Capacitance.

Solving this system of equations for Q results in an expression of the form,

$$\begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_n \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} \tag{2.16}$$

where $[c] = [p]^{-1}$ is referred to as the *generalized capacitance matrix*. Coefficients of the form c_{ii} represent the effective self-capacitance of the i^{th} conductor. The effective self-capacitance of a single conductor in a system of conductors is defined as the total charge on the conductor when it is maintained at unit potential and all other conductors are held at zero potential. Coefficients of the form c_{ij} where $i \neq j$ are referred to as *coefficients of induction*. These coefficients are always negative or zero and satisfy the relation $c_{ij}=c_{ji}$. They represent the ratio of the charge on the i^{th} conductor to the absolute potential of the j^{th} conductor when all conductors except the j^{th} conductor are at zero potential. Self and mutual capacitance values, which are always non-negative, can be calculated from the elements of the generalized capacitance matrix using the relations,

$$\begin{aligned} \mathrm{C_i} &= c_{i1} + c_{i2} + \cdots + c_{in} \\ \mathrm{C_{ij}} &= -c_{ij} \end{aligned} . \tag{2.17}$$

Once these values have been calculated for a system of conductors (e.g., Figure 2.6), the behavior of the system can be analyzed using simple circuit modeling techniques.

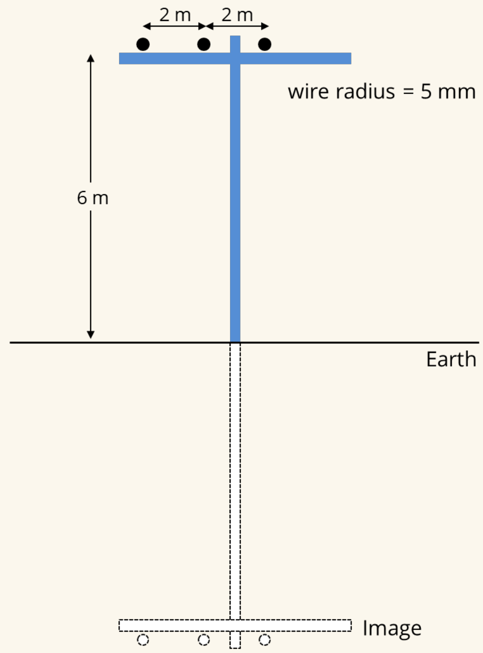
Example 2-5: Capacitance of Power Lines

Consider the power lines illustrated below. Three wires are mounted on poles $h = 6$ m above ground. The wire radius is $a = 5$ mm. Each wire is represented by a line charge at the wire's center.

The potential between 2 points at distances r_1 and r_2 from the uniform line charge is,

$$V_{12} = \frac{q}{2\pi\varepsilon_0} \ln\left(\frac{r_2}{r_1}\right)$$

where q is the magnitude of the line charge in coulombs per meter. If r_1 is set to the wire's radius, a, and $r2$ is set to twice the distance of the wire above ground, h, the potential on the surface of the i^{th} wire due to the wire's own charge and its image in the ground plane is,



$$V_i = \frac{q_i}{2\pi\varepsilon_0} \ln\left(\frac{2h}{a}\right) = p_{ii} q_i$$

where p_{ii} is a coefficient relating charge to potential as defined in (2.14). The potential on the j^{th} wire due to the charge on the i^{th} wire is readily shown to be,

$$V_{ji} = \frac{q_i}{2\pi\varepsilon_0} \ln\left(\frac{d'_{ij}}{d_{ij}}\right) = p_{ij} q_i$$

where d_{ij} is the distance between the i^{th} and j^{th} wire and d'_{ij} is the distance between the i^{th} wire and the image of the j^{th} wire.

Using the dimensions given in the figure, the following values are calculated,

$$p_{11} = p_{22} = p_{33} = \frac{1}{2\pi\varepsilon_0} \ln\left(\frac{2(6)}{0.005}\right) = 1.4\times 10^{11} \quad \text{m/F}$$

$$p_{12} = p_{21} = p_{23} = p_{32} = \frac{1}{2\pi\varepsilon_0} \ln\left(\frac{\sqrt{2^2+12^2}}{2}\right) = 3.25\times 10^{10} \quad \text{m/F}$$

$$p_{13} = p_{31} = \frac{1}{2\pi\varepsilon_0} \ln\left(\frac{\sqrt{4^2+12^2}}{4}\right) = 2.07\times 10^{10} \quad \text{m/F.}$$

Putting these results in matrix form gives,

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 1.4\times 10^{11} & 3.25\times 10^{10} & 2.07\times 10^{10} \\ 3.25\times 10^{10} & 1.4\times 10^{11} & 3.25\times 10^{10} \\ 2.07\times 10^{10} & 3.25\times 10^{10} & 1.4\times 10^{11} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix}.$$

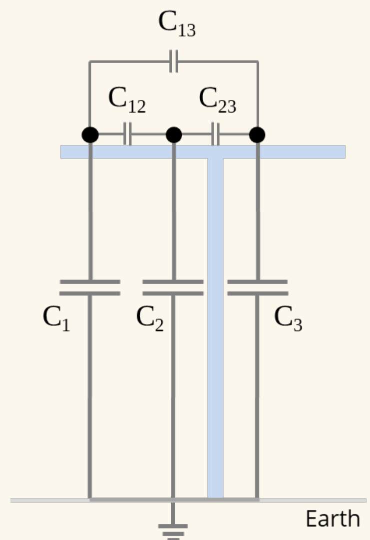
Inverting the matrix results in the following values for the generalized capacitance matrix,

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} 7.62\times10^{-12} & -1.59\times10^{-12} & -0.76\times10^{-12} \\ -1.59\times10^{-12} & 7.88\times10^{-12} & -1.59\times10^{-12} \\ -0.76\times10^{-12} & -1.59\times10^{-12} & 7.62\times10^{-12} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}.$$

We can then solve for the mutual capacitance between each wire and the wire-to-ground capacitances.

$$\mathrm{C}_1 = \mathrm{C}_3 = c_{11} + c_{12} + c_{13} = 5.27 \text{ pF/m}$$
$$\mathrm{C}_2 = c_{21} + c_{22} + c_{23} = 4.7 \text{ pF/m}$$
$$\mathrm{C}_{12} = \mathrm{C}_{23} = -c_{12} = 1.59 \text{ pF/m}$$
$$\mathrm{C}_{13} = -c_{13} = 0.76 \text{ pF/m}$$

These values can be used to calculate the capacitive coupling between the power lines using the equivalent circuit illustrated on the right.



In this example, the charge was uniformly distributed on the wire. However, when conductive objects are in close proximity, such as wires in a cable bundle or printed circuit board traces, charge may be concentrated on certain parts of the conductors as illustrated in Figure 2.7.

In these cases, an accurate closed form expression relating potential to the charge on the conductors may not be obtainable. The problem can still be solved, however, by breaking each conductor into a system of smaller conductors, each of which has an approximately uniform charge distribution. The self and mutual capacitances of each "piece" can then be calculated and the results recombined to get the self and mutual capacitances of the larger objects. This technique is efficient, accurate and relatively easy to apply using static field solvers. The concept of breaking a large problem down into a system of smaller problems is a very powerful one. A variety of electromagnetic modeling and analysis problems are solved using this general approach.

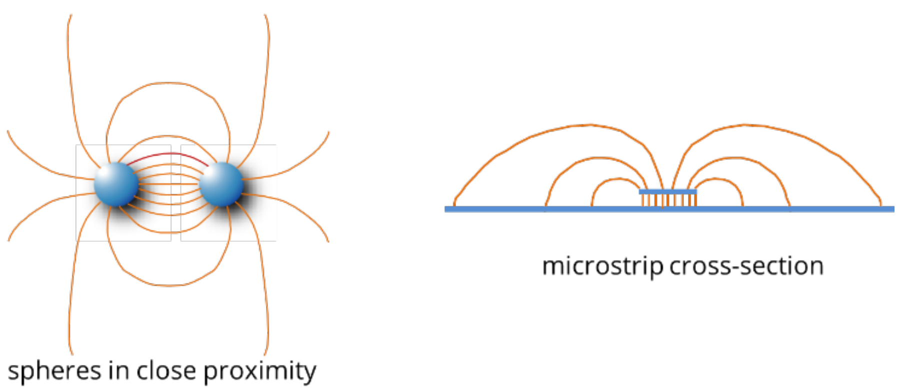


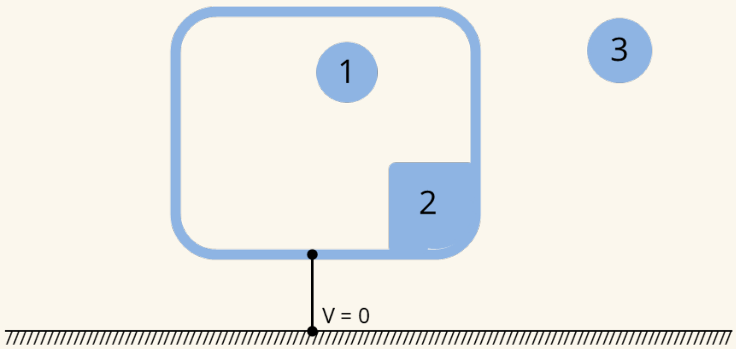
Figure 2.7. Geometries with non-uniform charge distributions.

Example 2-6: Coupling Between Shielded Conductors

Suppose a conductive object is surrounded by another conductor as illustrated below. The charge and absolute potential of this configuration are related by the system of equations,

$$q_1 = c_{11}V_1 + c_{13}V_3$$
$$q_2 = c_{21}V_1 + c_{23}V_3$$
$$q_3 = c_{31}V_1 + c_{33}V_3$$

since V_2 is our reference potential and is set to zero.



In the special case where $q_1 = 0$, the potential V_1 also must be zero because the potential everywhere inside a closed, equipotential surface without enclosed sources must equal the potential on the surface. Setting q_1 and V_1 equal to zero in the equation above indicates that,

$$c_{13}V_3 = 0$$

which must be valid for any value of V_3. Therefore, c_{13} must be zero.

Although this value was derived by setting $q_1 = 0$, the coefficients of induction are functions of the geometry only. Therefore $c_{13} = C_{13} = 0$ even if there is charge on the first conductor. In other words, a charge on the first conductor does not affect the potential of the third conductor. Also, because the capacitance matrix is symmetric, $c_{31} = C_{31} = 0$, and a charge on the outer conductor cannot affect the potential of the inner conductor.

The #2 conductor is an electrostatic shield. Static electric field and charge distributions outside the shield are completely independent of those inside the shield. This principle is often used to protect sensitive devices from strong external fields or to contain the fields of undesired sources.

While the definition of capacitance depends on the concept of electrostatic potential, values of capacitance are a function of the geometry. Therefore, capacitance also has meaning in time varying situations provided the conductive surfaces are small enough to ensure that the potential and charge on the surface are nearly constant. Generally, this condition is satisfied when all dimensions of the surface are much smaller than the shortest wavelength of interest.

Inductance

Capacitance was defined in terms of electric charge generating an electric field resulting in a voltage proportional to the charge. Inductance is a property associated with electric current that produces a magnetic field resulting in a coupled magnetic flux proportional to the current. In the same way that capacitance helps engineers to quantify electric-field coupling, inductance is useful for quantifying magnetic-field coupling.

A conductor of any shape can hold an electric charge, so conductors of any shape can have a self capacitance. But current flows in loops, so *self inductance* is a property of current loops. And just as the electrical field coupling two conductors can be quantified through as a mutual capacitance, the magnetic field coupling two current loops can be quantified in terms of a *mutual inductance*.

In the EMC literature, it's not uncommon to see references to the inductance of a wire, trace, via, or ground strap. When these things are part of a current loop, they contribute to the loop inductance. However, their contribution can only be determined when the rest of the loop is identified. Concepts such as *partial inductance* and *branch inductance* have been developed to help model geometries where the loop is not obvious or multiple loops exist. These concepts can be useful, but any application of inductance will depend on how the current loop is closed. In this section, we'll review a number of inductance concepts starting with the most basic.

External Inductance of a Wire Loop

Just as capacitance is a measure of the total electric flux that couples two conductive objects relative to their static potential, inductance is a measure of the total magnetic flux coupling two current paths relative to their steady-state current. A single wire loop has a self inductance just as a single conductive object has a self capacitance. Two wire loops can have a mutual inductance just as two conductive objects can have a mutual capacitance.

Figure 2.8 shows the lines of magnetic flux wrapping the wires in a current loop. Lines of magnetic flux form continuous loops that wrap around the source current. The direction of the flux lines can be determined by pointing the thumb of the right hand in the direction of the current. The fingers of the right hand will then curl in the direction of the magnetic flux created by the current. Integrating over all the flux lines passing through the loop, we could calculate the total magnetic flux *coupling* the loop as,

$$\Psi = \int_S \vec{B} \cdot \vec{ds} \text{ webers} \tag{2.18}$$

where the unit of magnetic flux is the *weber* (Wb), and $\boldsymbol{B}$ is the flux density in Wb/m^2. The unit of inductance is the *henry* (H), and the self inductance of the loop is,

$$L = \frac{\Psi}{I} \text{ henries.} \tag{2.19}$$

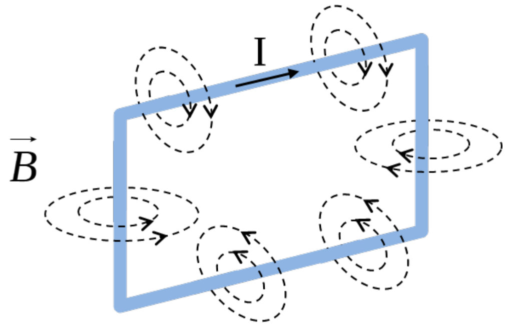


Figure 2.8. Magnetic flux coupling a current loop.

Since the amplitude of the magnetic flux will always be proportional to the current, the inductance, *L*, is not a function of current. *L* is a function of the loop geometry and material properties only.

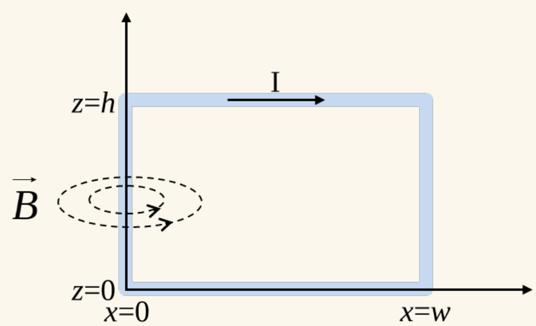
If the current increases in amplitude, the energy stored in the magnetic field must also increase. This added energy must come from the circuit, so the inductance of a circuit represents an impedance to an increasing current. Likewise, if the current decreases, energy stored in the magnetic field is absorbed back into the circuit which tends to prevent the current from decreasing. Therefore, inductance is a property of circuits that tends to resist changes in the amplitude of the current.

Example 2-7: Calculating the Inductance of a Rectangular Loop

Determine the inductance of a rectangular wire loop with height, h, and width, w.

Start by placing a uniform current, *I*, on the loop and calculating the total flux passing through the loop.

For this example, it is convenient to start with the Biot-Savart Law, which expresses the H-field due to an incremental current element, $I\,d\ell$.



$$\overrightarrow{dH} = \frac{I\overrightarrow{d\ell} \times \hat{a}_{12}}{4\pi R_{12}^2} \quad \text{A/m}$$

where *I* is the amplitude of the current, R_{12} is the distance from the current element to the field point, and $\hat{a}_{12}$ is a unit vector directed along the path of R_{12}.

To find the field at a point (r_0,ϕ_0,z_0) due to the side of the rectangular loop located on the z axis in the figure, we make the substitution $d\ell = zdz$ and integrate from $z=0$ to $z=h$

$$H_\varphi(r_0,0,z_0) = \int_0^h \frac{I\left(\hat{z}\times\hat{a}_{12}\right)}{4\pi R_{12}^2}\,dz \quad \text{A/m.}$$

For this problem, it is only necessary to solve for the ϕ component of $\vec{H}$, since this is the only component that penetrates the loop. Therefore, we are only interested in the radial component of $\hat{a}_{12}$, which is the cosine of the angle formed by the point $(r,0,z_0)$ and the z=0 plane. The cosine of this angle can be expressed as $\dfrac{r_0}{\sqrt{r_0^2+z_0^2}}$. Therefore,

$$H_\varphi\left(r_0,0,z_0\right)=\int_0^h \frac{I\,r_0}{4\pi\left[r_0^2+\left(z-z_0\right)^2\right]^{3/2}}\,dz \quad \text{A/m.}$$

Performing the integration and multiplying by the permeability of free space, μ_0, we obtain an expression for the $\hat{\phi}$ component of the magnetic flux density due to the first side of the loop,

$$B_\varphi\left(r_0,0,z_0\right)=\frac{\mu_0 I}{4\pi r_0}\left[\frac{h-z_0}{\sqrt{r_0^2+\left(h-z_0\right)^2}}+\frac{z_0}{\sqrt{r_0^2+z_0^2}}\right] \quad \text{Wb/m}^2.$$

To find the total flux penetrating the loop due to this segment, we integrate the flux density over the loop area assuming that the wire radius is much smaller than the height and width of the loop.

$$\begin{aligned}
\psi_{side1} &\approx \int_a^w\int_0^h B_\varphi\left(r_0,0,z_0\right)dz\,dr \\
&\approx \int_a^w \frac{\mu_0 I}{4\pi r_0}\int_0^h\left[\frac{h-z_0}{\sqrt{r_0^2+\left(h-z_0\right)^2}}+\frac{z_0}{\sqrt{r_0^2+z_0^2}}\right]dz\,dr \\
&\approx \int_a^w \frac{\mu_0 I}{4\pi r_0}\left[-2r+\frac{2\sqrt{h^2+r^2}}{r}\right]dr \\
&\approx \frac{\mu_0 I}{2\pi}\left[a-w+\sqrt{h^2+w^2}-h\ln\left(\frac{h+\sqrt{h^2+w^2}}{w}\right)-h+h\ln\left(\frac{2h}{a}\right)\right] \\
&\approx \frac{\mu_0 I}{2\pi}\left[-w+\sqrt{h^2+w^2}-h\ln\left(\frac{h+\sqrt{h^2+w^2}}{w}\right)-h+h\ln\left(\frac{2h}{a}\right)\right] \text{ webers.}
\end{aligned}$$

By symmetry, the total flux coupling the loop due to the segment on the opposite side of the loop is the same. An expression for the total flux coupling the loop due to the top and bottom segments is obtained by interchanging the *w* and *h* in the equation above. Summing the flux contributions from all four sides and dividing by the current, gives an expression for the self inductance of the loop,

$$L_{\text{rect loop}} \approx \frac{\mu_0}{\pi}\left[-2(w+h)+2\sqrt{h^2+w^2}-h\ln\left(\frac{h+\sqrt{h^2+w^2}}{w}\right)\right.$$
$$\left.-w\ln\left(\frac{w+\sqrt{h^2+w^2}}{h}\right)+h\ln\left(\frac{2h}{a}\right)+w\ln\left(\frac{2w}{a}\right)\right] \text{ henries.}$$

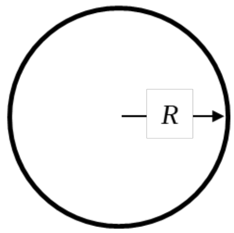
Although this is a simple loop, the closed form solution for the inductance is complex. To simplify it a little further, we can consider the case where h=w (i.e., a square loop),

$$L_{\text{square loop}} \approx \frac{2\mu_0 w}{\pi}\left[\ln\left(\frac{w}{a}\right)-0.774\right] \text{ henries.} \tag{2.20}$$

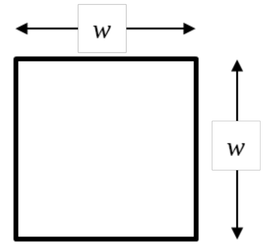
The size of the loop is the most important parameter affecting the inductance. For $w >> a$, the inductance is proportional to [w (ln w)]. The wire diameter also plays a role, especially for very thin wires. Note that a loop constructed of infinitely thin wire would have infinite inductance.

In the example above, we began our integration at the wire surface ($r = a$). In other words, we neglected any magnetic flux that may have existed within the wire itself. Inductance calculated using only the loop area external to the wire surface is called *external inductance*. At high frequencies, where the skin depth is a fraction of the wire diameter, most of the current flows near the surface of the wire and the flux inside the wire is nearly zero. In this case, the external inductance is usually the only inductance we need to calculate.

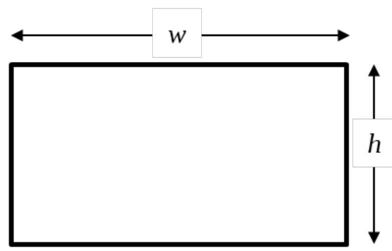
Figure 2.9 provides closed-form expressions for the external self inductance of five wire loop geometries. For transmission line geometries that are much longer in one dimension than the other two, it is generally much more accurate to calculate the inductance per unit length using the equations in Figure 2.10. The overall inductance of the loop can then be calculated by multiplying the inductance per unit length by the length of the transmission line.



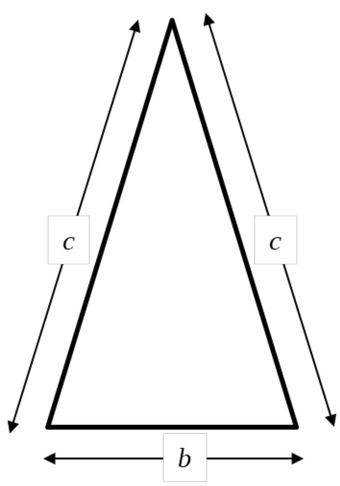
$$L_{circle} \approx N^2 R\mu \left[\ln\left(\frac{8R}{a} \right) - 2.0 \right] \quad \text{henries}$$

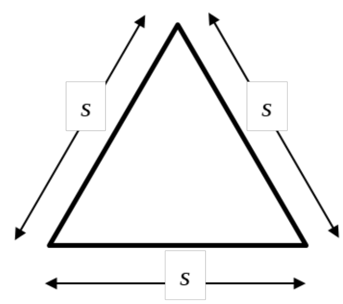


$$L_{square\ loop} \approx N^2 \frac{2\mu_0 w}{\pi} \left[\ln\left(\frac{w}{a} \right) - 0.774 \right] \quad \text{henries}$$



$$L_{rect\ loop} = N^2 \frac{\mu_0}{\pi} \left[-2(w+h) + 2\sqrt{h^2+w^2} - h\ln\frac{h+\sqrt{h^2+w^2}}{w} - w\ln\frac{w+\sqrt{h^2+w^2}}{h} + h\ln\frac{2h}{a} + w\ln\frac{2w}{a} \right] \quad \text{henries}$$

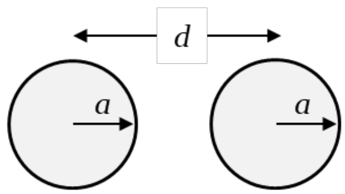




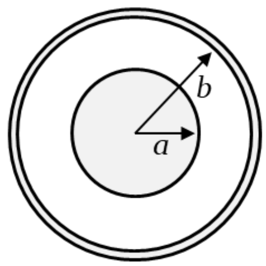
$$L_{equilateral\ triangle} \approx N^2 \frac{3\mu s}{2\pi} \left[\ln\left(\frac{s}{a} \right) - 1.405 \right] \quad \text{henries}$$

$$L_{isosceles\ triangle} \approx N^2 \frac{\mu}{2\pi} \left[2c\ln\left(\frac{2c}{a} \right) + b\ln\left(\frac{2c}{a} \right) - 2(b+c)\sinh^{-1}\frac{b^2}{\sqrt{4b^2c^2-b^4}} - 2c\sinh^{-1}\frac{2c^2-b^2}{\sqrt{4b^2c^2-b^4}} - (2c+b) \right] \quad \text{henries}$$

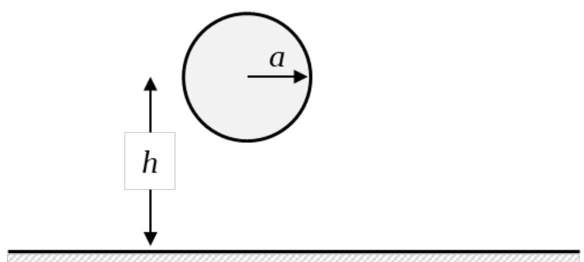
Figure 2.9. External inductance of round wire geometries
(wire radius = a, number of turns = N).



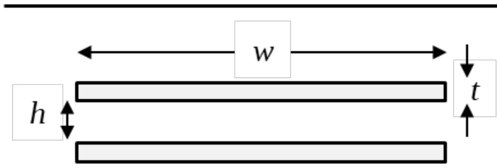
$$L_{\text{wire pair}} \approx \frac{\mu}{\pi} \cosh^{-1} \frac{d}{2a} \quad \text{H/m}$$



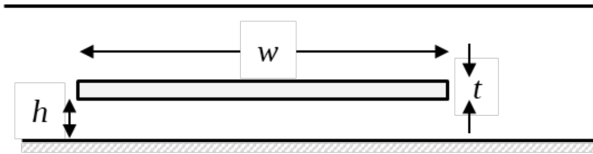
$$L_{\text{coaxial cable}} \approx \frac{\mu}{2\pi} \ln \frac{b}{a} \quad \text{H/m}$$



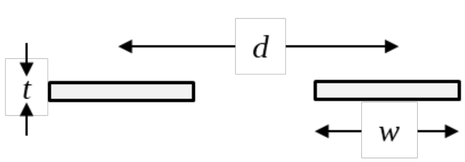
$$L_{\text{wire over plane}} \approx \frac{\mu}{2\pi} \cosh^{-1} \frac{h}{a} \quad \text{H/m}$$



$$L_{\text{parallel planes}} \approx \frac{\mu h}{w} \quad \text{H/m} \qquad (w \gg h, \;\; h > t)$$



$$L_{\text{wide trace over plane}} \approx \frac{\mu h}{w} \quad \text{H/m} \qquad (w >> h, \;\; h >> t)$$



$$L_{\text{coplanar traces}} \approx \frac{\mu}{\pi} \cosh^{-1} \frac{d}{w} \quad \text{H/m} \qquad (w > t)$$

Figure 2.10. External inductance per unit length of transmission line geometries.

Internal Inductance

At low frequencies where the diameter of a wire is much less than a skin depth, the current distribution within the wire is nearly uniform. This means that there is magnetic flux within the wire that couples the loop and contributes to the overall inductance. Calculating the exact contribution of this internal flux can be complicated because both the amount of flux and the loop area are functions of position within the wire. However, for wires with a circular cross section, it is relatively simple to calculate the contribution of the internal flux based on an energy definition of inductance.

Example 2-8: Calculating the Internal Inductance per Unit Length of a Round Wire

The energy stored in an inductance is equal to ½ LI^2. The energy stored in a magnetic field is equal to the energy density of the field, ½ μH^2, integrated over the entire volume of the field. Using these expressions, we can equate the energy stored by the internal inductance of a wire to the energy stored in the magnetic field within the wire,

$$\frac{1}{2} L_{\text{wire-internal}}\, I^2 = \int_v \frac{\mu}{2} \left| \vec{H} \right|^2 dv \quad \text{(J)}$$

$$= \frac{\mu}{2} \int_0^{\ell} \int_0^{2\pi} \int_0^a \left| \frac{Ir}{2\pi a^2} \right|^2 r\, dr\, d\varphi\, dz$$

where μ is the permeability of the wire and ℓ is the length of the wire.

Solving for $L_{wire\text{-}internal}$, we obtain,

$$L_{\text{wire-internal}} = \frac{\mu}{8\pi} \ell \quad \text{henries.}$$

Note that the expression for the internal inductance of a round wire is independent of the radius of the wire. For copper or aluminum wires, $\mu = \mu_0$ and the internal inductance per unit length is approximately $\mu_0/8\pi$ or 50 nH/m.

At low frequencies where the skin depth is much greater than the wire diameter, the total inductance of a wire loop is equal to the external inductance plus the internal inductance. Therefore, we can get the total self inductance of a square loop by combining Eq. (2.20) and the expression above for the internal inductance to get,

$$L_{\text{square loop}} \approx \frac{2\mu_0 w}{\pi} \left[\ln\left(\frac{w}{a} \right) - 0.774 \right] + 4w \frac{\mu}{8\pi}$$

$$\approx \frac{2\mu_0 w}{\pi} \left[\ln\left(\frac{w}{a} \right) - 0.774 + \frac{\mu_r}{4} \right] \quad \text{henries}$$

where μ_r is the relative permeability of the wire.

Note that the effect of the internal inductance is relatively minor for loops where $w >> a$. At higher frequencies, where inductance is more likely to be a concern, the internal inductance of the wire is even smaller due to the skin effect and the total inductance is approximately equal to the external inductance. The concept of internal inductance is an important one for an engineer to be aware of, but it can often be neglected when solving for the inductance of practical configurations.

Mutual Inductance

Just as two conducting objects can have a mutual capacitance, two current loops can have a mutual inductance. Figure 2.11 helps to illustrate the concept of mutual inductance. Two wire loops are shown. A current in the first loop generates a magnetic flux. Some of this flux passes through (or links) the second loop. The mutual inductance is defined as the ratio of the total magnetic flux that links the 2nd loop divided by the current in the 1st loop,

$$L_{21} = \frac{\Psi_{21}}{I_1} \text{ henries.} \tag{2.21}$$

If loop 2 consists of a single turn of wire (i.e., no flux lines can pass through the loop twice), then Ψ_{21} must be some fraction of the total flux generated by loop 1, Ψ_{11}.

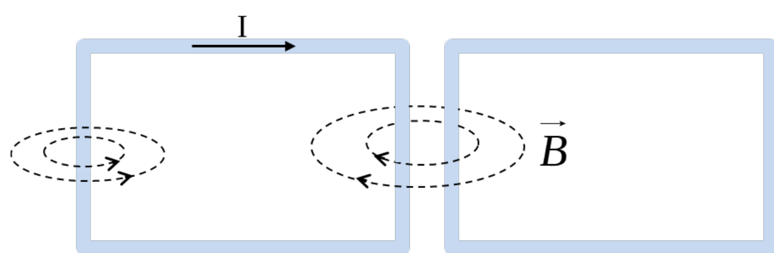


Figure 2.11. Mutual inductance in a pair of current loops.

The coupled flux, Ψ_{21}, can be written as the integral of the coupled flux density over the surface of the 2nd loop,

$$L_{21} = \frac{\int_{S2} \vec{B}_1 \cdot \vec{ds}}{I_1} \text{ henries.} \tag{2.22}$$

Applying Stokes's theorem, we can convert the surface integral above to a line integral,

$$L_{21} = \frac{\int_{S2} \left(\nabla \times \vec{A}_1 \right) \cdot \vec{ds}}{I_1} = \frac{\oint \vec{A}_1 \cdot \vec{d\ell_2}}{I_1} \text{ henries} \tag{2.23}$$

where $\vec{A}_1$ is the magnetic vector potential due to the current in loop 1. This vector potential can be expressed as a function of the current in loop 1 as follows,

$$\vec{A}_1 = \oint \frac{\mu I_1}{4\pi R} \vec{d\ell} \text{ webers/m} \tag{2.24}$$

where R is the distance between the current element, $d\ell$, and the field point. Combining Equations (2.23) and (2.24) we get an expression for the mutual inductance that is independent of the current,

$$L_{21} = \frac{\mu}{4\pi} \oint \oint \frac{\overrightarrow{d\ell_1} \cdot \overrightarrow{d\ell_2}}{R} \text{ henries.} \tag{2.25}$$

It is clear from this expression that $L_{21} = L_{12}$ since the integration in loops 1 and 2 can be taken in either order.

Another useful expression for the mutual inductance between two loops with self inductances, L_{11} and L_{22} is,

$$L_{21} = k\sqrt{L_{11} L_{22}} \text{ henries} \tag{2.26}$$

where $0 \leq k < 1$ depends on the fraction of the total flux that couples both loops. From this expression, it is clear that the mutual inductance cannot exceed the self inductance of the larger loop.

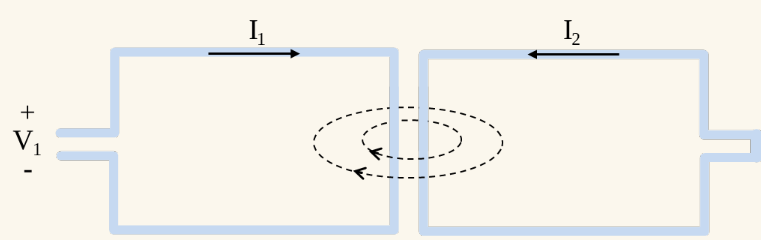
Effective Inductance

The term *effective inductance* is often used to describe the inductance that would be measured between two terminals in a system. If the measured impedance across these terminals is a positive reactance (i.e., imaginary and positive) at one particular frequency, then it looks like an inductance at that frequency. If the measured impedance is also proportional to frequency over a given frequency range, then it looks like an inductance over that entire range of frequencies.

Unlike self or mutual inductance, the concept of effective inductance can be applied to geometries other than loops. For example, we can define and calculate the effective inductance of a power supply output or a printed circuit board via. However, it is important to recognize that effective inductance generally depends on many factors and may only apply over a limited frequency range.

Example 2-9: Calculating the Effective Inductance of a Pair of Circuits

Two wire loops exist side by side as illustrated below. Both loops are rectangular and have a self inductance of 100 nH. The mutual inductance between the two loops is 20 nH. What is the effective inductance as viewed from the input terminals of the first loop?



We can start by defining the voltage and current in the first loop as V_1 and I_1, respectively. The voltage and current in the second loop are V_2 and I_2 and we can write the system of equations for the voltages and currents as follows,

$$V_1 = j\omega L_{11} I_1 \pm j\omega L_{12} I_2$$

$$V_2 = j\omega L_{21} I_1 \pm j\omega L_{22} I_2$$

The $\pm$ in these equations indicates that the flux generated by a current in loop 2 can either add-to or subtract-from the flux generated by the current in Loop 1 depending on the orientation of the loops. In this case, current induced in Loop 2 by the current in loop 1 will generate a flux that opposes the flux in Loop 1, so the minus sign is the correct choice.

Since the second loop is shorted, $V_2 = 0$ and substituting into the second equation above yields,

$$I_2 = \frac{L_{21}}{L_{22}} I_1 = \frac{20 \text{ nH}}{100 \text{ nH}} I_1 = 0.20 I_1$$

Therefore, the equation above for V_1 can be rewritten,

$$V_1 = j\omega(100 \text{ nH}) I_1 - j\omega(20 \text{ nH})(0.2) I_1 = j\omega(96 \text{ nH}) I_1$$

In other words, the effective inductance at the terminals of Loop 1 is 96 nH.

In the above example, the effective inductance could be used to model the input impedance at the terminals of the first loop at any frequency where the loop dimensions and the distance between the loops was small relative to a wavelength. At higher frequencies, the input impedance could not be modeled over a broad band of frequencies with a single inductor.

Partial Inductance

In the previous section, we showed that the mutual inductance between two loops n and m can be expressed as,

$$L_{nm} = \frac{\mu}{4\pi} \oint \oint \frac{\overrightarrow{d\ell_n} \cdot \overrightarrow{d\ell_m}}{R_{nm}} \tag{2.27}$$

where R_{nm} is the distance between the elements $\overrightarrow{d\ell_n}$ and $\overrightarrow{d\ell_m}$. Each infinitesimal element of current in loop n generates a magnetic flux that potentially "couples" to every infinitesimal current element in the other loop. Summing over all of these interactions yields the mutual inductance between the two loops. Setting n=m, it is possible to calculate the self inductance of a loop using this same equation.[3]

If we were to break the loop into a finite number of segments, we could write (2.27) as a sum of *partial inductances*,

$$L_{ij} = \frac{\mu}{4\pi} \sum_{i=0}^{I} \sum_{i=0}^{J} \ell_{ij} \tag{2.28}$$

[3] By setting all of the source points to be in the center of the wire and the observation points on the wire surface, R_{nm} will never equal zero and the singularity in the integral is avoided.

where $l_{ij} = \int\limits_{\text{segment i}} \int\limits_{\text{segment j}} \frac{\overrightarrow{d\ell_i} \cdot \overrightarrow{d\ell_j}}{R_{ij}}$.

The quantity ℓ_{ij} is referred to as the *partial mutual inductance* between segments i and j.

The quantity ℓ_{ii} is similarly referred to as the *partial self inductance* of segment i.

The concept of partial inductance can be utilized by computer modeling tools to calculate the inductance of complex configurations. Computer modeling techniques that employ both partial inductance and partial capacitance concepts are capable of performing complex electromagnetic simulations. However, it is important to remember that the partial self inductance of a wire segment, via or trace is not a particularly useful number by itself. It does not necessarily relate directly to any measurable property of the physical circuit.

Branch Inductance

Although the concept of partial inductance cannot normally be applied to individual pieces of a circuit, it is often convenient to refer to the contribution that each "part" of a current loop makes to the overall inductance of the loop. For example, when calculating the inductance of the square wire loop, we determined the contribution that each of the 4 sides made to the total inductance and added them together. In the case of the square loop, all 4 sides contributed equally, however we could apply this same approach to nearly any circuit configuration with a well-defined current path.

Consider the loop shown in Figure 2.12. A wire half-loop extends above the surface of a finite sized plane. Current flowing in the wire returns to its source through the plane. The height of the half-loop is small relative to the length of the half-loop or the width of the plane.

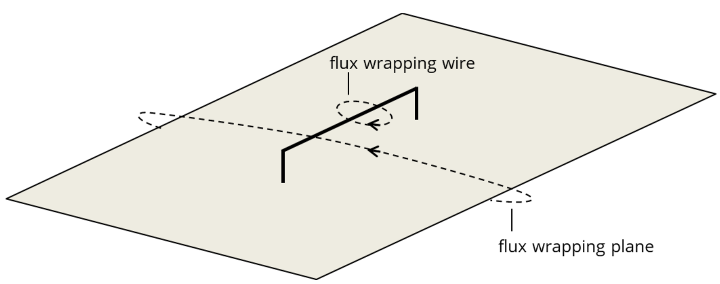


Figure 2.12. A wire half-loop above a plane.

Applying image theory, the magnetic flux coupling the loop due to the current in the wire can be calculated using the method in Example 2-7 for determining the inductance of a rectangular loop. The *branch inductance* of each piece of the loop is defined as the flux coupling the loop that is generated by the current in that piece divided by the magnitude of the current. Summing the branch inductances of every piece of a loop yields the self inductance of the loop.

Branch inductance is a convenient way of expressing the contribution that one part of a circuit makes to the total inductance. For the half-loop in Figure 2.12, most of the flux in the loop is due to the current in the horizontal wire. The vertical wires contribute less flux due to their relatively short length. The plane contributes little because the magnetic field must wrap all the way around its width significantly reducing the amplitude of the flux. Since the inductance of the horizontal wire accounts for most of the total inductance, changes to that wire have a much more significant effect on the loop inductance than changes to the other parts of the circuit.

It is not uncommon to see the term *partial inductance* used to describe a branch inductance. For example, one might refer to the partial inductance of a via, trace or plane in a signal path. In most cases, the intent is clear, and the choice of terms is not a problem. However, it's important to recognize that this partial inductance and the partial inductance defined in the previous section are two very different quantities. Calculations of the partial inductance as defined in the previous section cannot be used to determine the branch inductance of a via, trace or plane.

Inductance of a Wire or Ground Strap

By now it should be clear that inductance is a property of loops and that a wire, ground strap, or any conductor that is not part of a well-defined loop does not have a well-defined inductance. Nevertheless, the inductance of components like ground straps and vias can be extremely important when modeling high-frequency circuits and systems. It can't be ignored, just because it is not well-defined.

Fortunately, the branch inductance of these components can often be estimated even when the rest of the loop has not been precisely identified. To see how this works, consider the four wire loops illustrated in Figure 2.13. These loops are shown approximately to scale relative to one another. The first loop is circular with a loop radius of only 3 mm. The wire radius is 0.5 mm. The second loop is also circular with a loop radius of 3 cm and a 1-mm wire radius. The third loop is square but has the same circumference as the second loop. The fourth loop is also square, but the sides are 5 times larger (loop area 25 times larger) than the third loop. The calculated inductances for each loop are shown as well as the inductance divided by the length of the wire (loop perimeter).

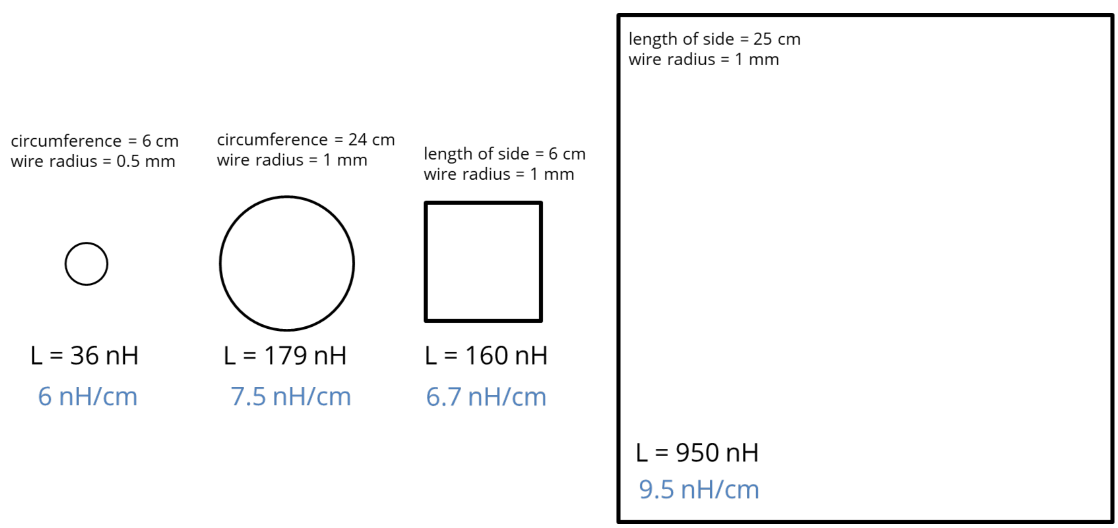


Figure 2.13. Wire loop geometries.

Note that even though the loop sizes vary dramatically, the inductance per unit length is on the order of 10 nH/cm. This is a useful approximation to keep in mind. As long as the return path is not too close and not too far, multiplying the length of a wire or strap by 10 nH/cm yields a reasonable order-of-magnitude estimate of its branch inductance.

We can make this approximation a little more accurate by accounting for the wire or strap cross-sectional dimensions. Consider the square loop formed by connecting four identical ground straps as shown in Figure 2.14. The method described in Example 2-7 can be used to calculate the inductance of a square loop formed with round wire. These ground straps are not round wires, but we can use the same equation by recognizing that flat rectangular conductors with a width, w, have an effective radius, $a_e = 0.25\,w$.

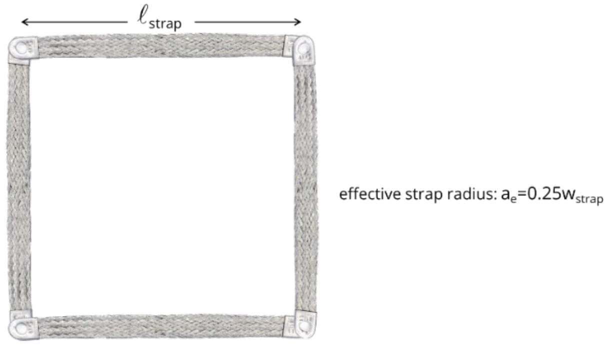


Figure 2.14. Square loop formed from ground straps.

Applying (2.20) to the calculate the inductance of the loop in Figure 2.14, we get

$$L = \frac{2\mu_0 \ell_{strap}}{\pi}\left[\ln\left(\frac{4\ell_{strap}}{w_{strap}}\right) - 0.774\right]. \tag{2.29}$$

This can then be divided by the circumference of the loop to get the inductance per unit length.

$$\begin{aligned}
\frac{L}{4\ell_{strap}} &= \frac{\mu_0}{2\pi}\left[\ln\left(4\left[\frac{\ell_{strap}}{w_{strap}}\right]\right) - 0.774\right] \\
&= \frac{\mu_0}{2\pi}\left[\ln(2) + \ln\left(\frac{2\times\ell_{strap}}{w_{strap}}\right) - 0.774\right] \\
&\approx 2\times10^{-7}\left[\ln\left(\frac{2\times\ell_{strap}}{w_{strap}}\right)\right]\text{henries/m} \\
&\approx 20\left[\ln\left(\frac{2\times\ell_{strap}}{w_{strap}}\right)\right]n\text{H/cm.}
\end{aligned} \tag{2.30}$$

Multiplying the result in (2.30) by the length of the strap yields the branch inductance of one strap in this loop,

$$L \approx 20\ell_{strap}\left[\ln\left(\frac{2\times\ell_{strap}}{w_{strap}}\right)\right]n\text{H.} \tag{2.31}$$

We would have obtained a similar result if the loop had been any other relatively large open shape. For this reason, it is convenient to use (2.31) to estimate the branch inductance of ground straps used in applications where the rest of the loop is undefined, but reasonably large.

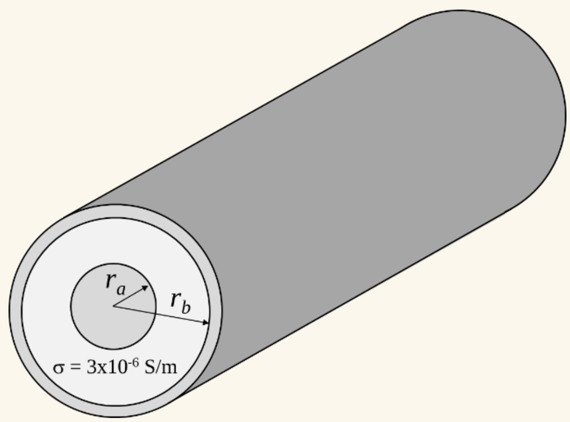
Conductance

The term conductance can refer to any inverse resistance, G=1/R. However, it is generally applied to imperfect dielectrics in the same way that we use resistance to describe imperfect conductors. It is the ratio of the current flowing in the dielectric between two conductors to the potential difference between those conductors.

Example 2-10: Conductance per Unit Length of a Coaxial Cable

Determine the conductance per unit length of a coaxial cable with an inner conductor radius, r_a, and an outer conductor radius, r_b.

Suppose that the dielectric between the two coaxial conductors illustrated below is lossy and has a conductivity, $\sigma = 3\text{x}10^{-6}$ S/m. If we assume a uniform current, I, flows between the two conductors, then the magnitude of the current density, $\vec{J}$, decreases as it moves from the inner conductor to the outer conductor because it is spread over an increasing area. By dividing the total current by this area, we obtain an expression for the current density,



$$\vec{J} = \frac{I}{2\pi r \ell}\hat{r} \quad \text{A/m}^2$$

where r is the radial distance from the center of the cable and ℓ is the length of the cable.

The electric field is found by applying Ohm's Law,

$$\vec{E} = \frac{\vec{J}}{\sigma} = \frac{I}{2\pi r \ell \sigma}\hat{r} \quad \text{V/m} .$$

and the voltage between the two conductors is found by integrating the electric field,

$$V_{ab} = \int_{r_a}^{r_b} \frac{I}{2\pi r \ell \sigma} dr = \frac{I}{2\pi r \ell \sigma} \ln\left(\frac{r_b}{r_a}\right) \quad \text{volts.}$$

The conductance per unit length is the ratio of the current per unit length to the voltage,

$$G = \frac{2\pi\sigma}{\ln\left(\frac{r_b}{r_a}\right)} \quad \text{S/m.}$$

Note the similarity between the expression for the conductance above and the expression for the capacitance per unit length of a coaxial cable,

$$C = \frac{2\pi\varepsilon}{\ln\left(\frac{r_b}{r_a}\right)} \quad \text{F/m.} \tag{2.32}$$

The procedure for determining the capacitance between conductors is analogous to the procedure for determining conductance. Instead of beginning with an electric flux due to static charges, we begin with an electric current. Instead of a variable, ε, which is the ratio of the electric flux to electric field, we use the variable, σ, which is the ratio of electric current to electric field. Generally, any procedure used to find the capacitance of a configuration can also be used to determine the conductance by making these substitutions of variables.

Electronic Components

What's the difference between a *resistance* and a *resistor*? Resistance is the term we use to describe the real part of the ratio of voltage across a device to the current through a device. Resistors are devices that we use in actual circuits to provide a given amount of resistance.

Resistors have resistance. However, as every good EMC and signal integrity engineer knows, resistors in a circuit can also have significant inductance and/or capacitance. Figure 2.15 illustrates the magnitude of the measured impedance for a typical 50-Ω resistor with axial leads as a function of frequency.

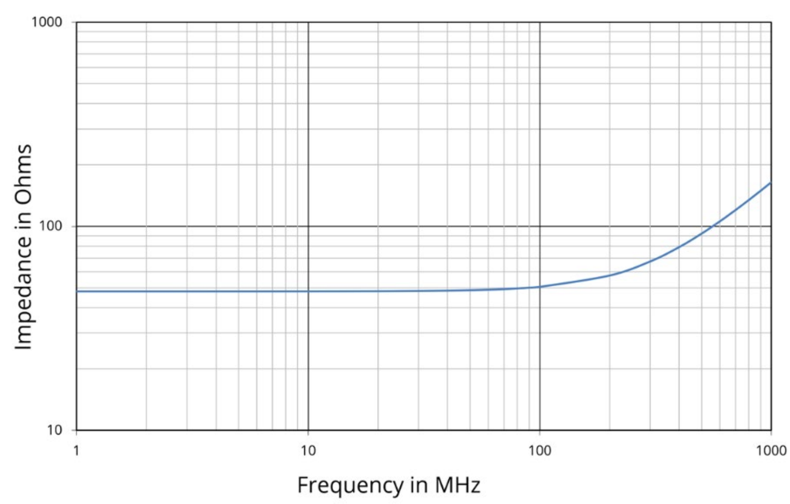


Figure 2.15. Impedance of a 50-Ω resistor with 25 nH of connection inductance.

At frequencies below 100 MHz, the magnitude of the impedance is independent of frequency and has a value within 15% of the nominal resistance. However, above 200 MHz the impedance increases with frequency much like you would expect from an inductor. A simple circuit that models the behavior of the resistor is shown in Figure 2.16. The resistance in this circuit is close to the nominal value. However, the current flowing through the resistor and its connecting leads generates a magnetic flux that couples the circuit introducing a small inductance in series with the resistance. Also, the voltage across the end caps creates electric field lines within and external to the resistor package. At higher frequencies or for larger resistances, displacement current associated with these field lines bypasses the resistive, conduction current path and is represented by a capacitance in parallel with the resistance.

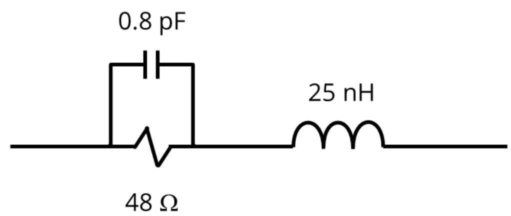


Figure 2.16. Equivalent circuit for an axial lead resistor.

The inductance and capacitance in this resistor model are called *parasitic* components because they represent unintentional properties of the component. If we were to try to model the impedance of this resistor at higher frequencies, additional parasitic values could be introduced. However, the model in Figure 2.16 is generally sufficient to describe the behavior of most resistors within their useful frequency range.

Figure 2.17 shows the measured impedance of a 0.01-μF surface-mount capacitor. Once again, at low frequencies the impedance is what one would expect from a capacitor. However, above 100 MHz, the impedance increases proportional to the frequency as if the device under test were an inductor.

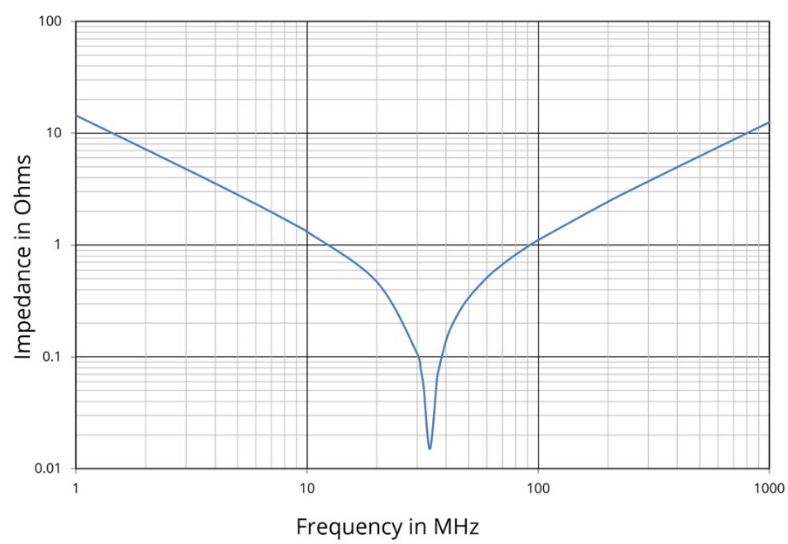


Figure 2.17. Measured impedance of a 0.01-μF surface mount capacitor.

An equivalent circuit for this capacitor is shown in Figure 2.18. In this circuit, the L represents the parasitic inductance associated with the magnetic flux coupling the capacitor's package and connections to the rest of the circuit. The R represents the resistive loss in the package and connections.

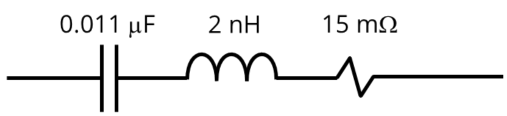


Figure 2.18. Equivalent circuit for a surface mount capacitor.

While the value of L will change significantly depending on how the capacitor is connected to the circuit, the value of R is normally dominated by losses within the package. This value (the Equivalent Series Resistance or ESR), is often measured at specific frequencies included in the capacitor specifications.

Inductors also have parasitic values. Figure 2.19 illustrates a plot of the impedance of a 5-μH surface mount inductor. The equivalent circuit model is shown in Figure 2.20.

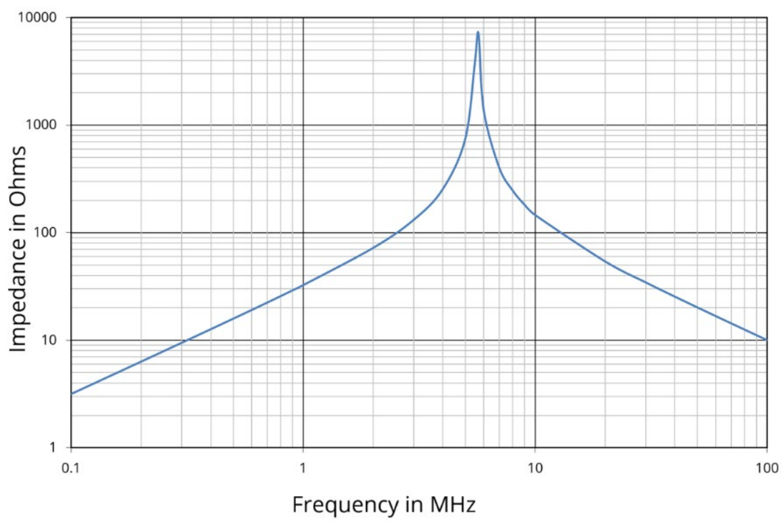


Figure 2.19. Impedance of a 5-μH surface mount inductor.

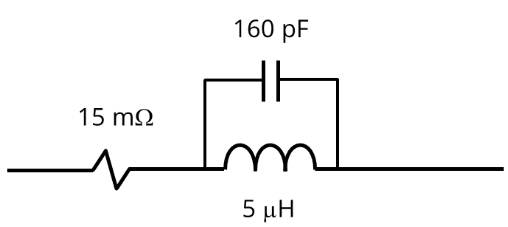


Figure 2.20. Equivalent circuit for a surface mount inductor.

The parasitic capacitance represents electric field coupling between windings and/or between the end caps of the package. Unless lossy ferrites are employed, the parasitic resistance is primarily associated with the conductive loss in the windings.

Types of Resistors

Metal film

By far the most common type of resistor in use today is the metal film resistor. These resistors are manufactured by etching a thin layer of metal on an insulating substrate as illustrated in Figure 2.21. The resistance and power rating are controlled by varying the length and width of the metal film trace.

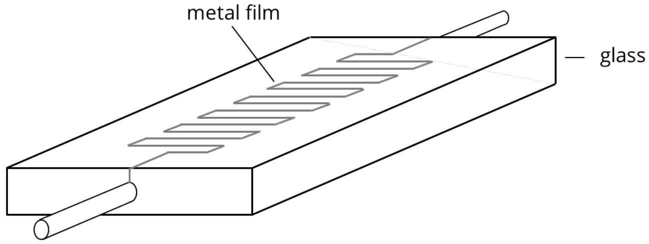


Figure 2.21. Metal film resistor construction.

The advantage of metal film resistors is that the resistance value can be controlled fairly precisely. They are also generally the least expensive option for low to moderate power applications. A primary disadvantage of these resistors is that the metal film can be vaporized by current transients whose peak power briefly exceeds the resistor's power rating.

Carbon Composite

Carbon composite resistors are made by packing a conductive powder or paste between the two end plates of the resistor package. An advantage of this type of resistor is that it is not particularly susceptible to current transients. The peak power that these resistors can absorb is generally much higher than their steady state power rating. Resistors of this type are well suited for applications where the components are likely to encounter voltage or current spikes.

Wire wound

Wire wound resistors consist of a lossy insulated wire wound in a helical shape around a ceramic core. The core helps to pull heat away from the wires. These resistors are used in applications where the resistors are required to dissipate a lot of power. Their main advantage is their power handling capability. However, they typically cost more than other types of resistors and they tend to have a significant amount of parasitic inductance.

Types of Capacitors

Ceramic

Look at any densely populated printed circuit board and you will see a large number of capacitors. On most boards, the majority of these will be ceramic capacitors consisting of pairs of metal plates with a ceramic dielectric material between them. Ceramic capacitors are relatively inexpensive and reliable. They can be divided into two classes depending on the characteristics of their ceramic dielectric. Class 1 capacitors (e.g., C0G and NP0 capacitors) have relatively precise values that are stable with changes in voltage and temperature. Class 2 capacitors (e.g., X5R and X7R capacitors) have higher permittivity dielectrics, so larger values can be obtained in a given package size. Class 2 capacitors have capacitance values that can vary significantly depending on the applied voltage or temperature.

Electrolytic

Electrolytic capacitors employ a special dielectric material that has a higher permittivity than ceramic materials, but only in one polarity. They are only used in situations where there is a DC bias voltage. The higher permittivity allows larger values of capacitance to be obtained in a given package size. Generally, the larger capacitors on a printed circuit board (physically and by nominal value) are electrolytics. Their main advantage is their large nominal value. However, they do not exhibit the same temperature and frequency stability as other types of capacitors.

Tantalum

Tantalum capacitors are a special type of electrolytic capacitor made with electrodes that contain the element tantalum. Like other electrolytic capacitors, they can pack a lot of capacitance into a small package. However, tantalum capacitors tend to have better temperature and frequency characteristics than other electrolytics. The main disadvantages of this type of capacitor are their relative cost and the fact that they are easily damaged by voltage transients.

Film

Film capacitors employ an insulating plastic film as the dielectric. The film may be sandwiched between two layers of metal foil, or the metallization may be vacuum-deposited on the surface of the film. Film capacitors generally have larger package sizes than ceramic or electrolytic capacitors with the same nominal capacitance. However, they tend to be more stable and reliable than electrolytic capacitors, and they are non-polarized. Film capacitors are typically better suited for high-energy applications than ceramic capacitors due to their physical size and capacity to carry large currents.

High-Voltage

Capacitors designed to withstand very high voltages employ dielectric materials such as mica that can withstand strong electric fields without breaking down. They may also utilize more electrodes with greater spacing. Their primary disadvantages are their cost and their relatively large package size.

Super-Capacitors

Super capacitors (also called ultra-capacitors or electric double layer capacitors) employ a technology that uses the space between small conductive particles to store the electric field energy. Rather than traditional plates, the electrodes in a super-capacitor are formed by a chemical reaction. As a result, these capacitors can have extremely high nominal values (as much as several Farads or more) in a relatively small package. They are often used to replace back-up batteries in electronic equipment. They are relatively expensive, and they can be dangerous if their terminals are accidentally shorted.

Types of Inductors

Ferrite Core

The most common type of inductor found in electronic circuits consists of a thin wire wound around a ferrite core. The ferrite material has a high permeability which increases the amount of inductance for a given loop area and number of turns. This permits higher values of inductance in smaller packages than one could obtain without the ferrite material. However, for high values of current, it is possible to saturate the ferrite material causing its effective permeability to drop significantly. This non-linear behavior can result in the creation of signal harmonics and other problems.

Shielded inductors surround the windings with ferrite. This helps to contain the spread of magnetic field lines and reduces the likelihood of unwanted coupling to nearby components and circuits.

Air Core

Air core inductors consist of wire wrapped around an air (or any non-ferrous) core. They are employed in situations like high-frequency amplifier designs where linear behavior is critical or in circuits such as power circuits where the inductor must be able to handle high currents.

Identifying Current Paths

Electrical engineers are generally more comfortable thinking of electrical signals in terms of voltage rather than current. Digital logic levels are normally determined by signal voltages and power supplies are generally constant voltage sources. Voltages in a circuit can usually be measured using simple probes without loading the circuit significantly.

Currents, on the other hand, are more difficult to measure. Typically, current is measured by passing it through a small resistance and measuring the voltage dropped across the resistance. Alternatively, we measure the voltage induced in a loop by the magnetic field accompanying the current. In many circuit designs, the maximum current is specified but little attention is paid to current waveforms or current paths.

One of the most important skills that an EMC engineer must develop is the ability to identify and locate both the intentional and unintentional currents in an electronic system. Current is primarily responsible for three of the four possible EMC coupling mechanisms described in the first chapter. Without understanding how and where the currents in each circuit flow, it can be difficult to anticipate problems in new designs or fix problems in existing designs.

Quiz Question

The ultimate destination of the signal current that flows out of a pin in an integrated circuit is,

a.) earth ground
b.) the ground plane of the printed circuit board
c.) one or more of the other pins on the integrated circuit

The first rule to remember when identifying the path of a current is that **all currents return to their source**. In other words, currents flow in loops. Yes, there are displacement currents (i.e., time-varying fields that result when the net charge on a conductor changes). However, net charge cannot be created or destroyed and the current flowing out of one device pin must equal the current flowing in on the other pins[4].

Digital circuit designers often neglect to consider where the currents in their designs will flow. It is not uncommon to see the current path from the signal source to the load carefully laid out while the path from the load back to the source is left to chance.

Many years ago, EMC engineers at IBM were evaluating a product that had severe electromagnetic susceptibility problems. The system employed an 8-bit communication bus that was routed on a cable between two boxes. When the EMC engineers examined the cable, they found that it had exactly 8 wires (one for each signal, but none for returning the signal current). The product engineer explained that the signals were voltages referenced to the chassis ground of each box. What the product engineer didn't realize was that the signal return currents had to flow through the chassis, then the power cable, then through the building wiring, then through the power cable and chassis of the source box. This relatively high-impedance path caused the chassis of the two boxes to be at different potentials. In addition, the large loop area associated with the signal current path was capable of picking up significant amounts of electromagnetic noise.

However, this was not the whole story. As illustrated in Figure 2.22, the chassis/building ground was one possible signal current return path, but it was not the only one. In this case, the current in any signal wire also had the option of returning to the source through the other signal wires. For example, suppose in this case that a logic “1” was represented by a positive 5-volts on the signal line and a logic “0” was 0 volts. Then at any given time the current from the logic “1” lines could return to the source through the logic “0” lines. In order for this to happen, the current from logic “1” lines would flow through their own load resistances and then through the load resistances of the logic “0” lines inducing a negative voltage across these loads.

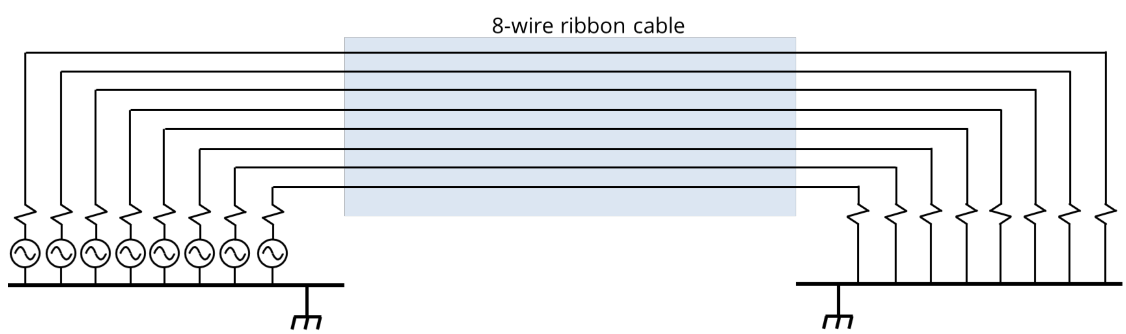


Figure 2.22. An 8-bit data bus with no explicit signal return path.

Whether the currents would return to their respective sources through the chassis ground or through the other signal lines would depend on the relative impedance of these two options. The second rule to apply when identifying the path of a current is that **current favors the path(s) of least impedance**.

Consider the configuration illustrated in Figure 2.23. A variable frequency source places a voltage across the input to a coaxial cable. Signal current flows along the inner conductor of the coaxial cable then through the resistor. At that point there are two possible paths that

[4] Assuming there is negligible displacement current to/from the device package.

the current can take to return to the source. Current can flow back along the shield of the cable, or it can take a short cut through a short, wide copper strap attached to both ends of the cable.

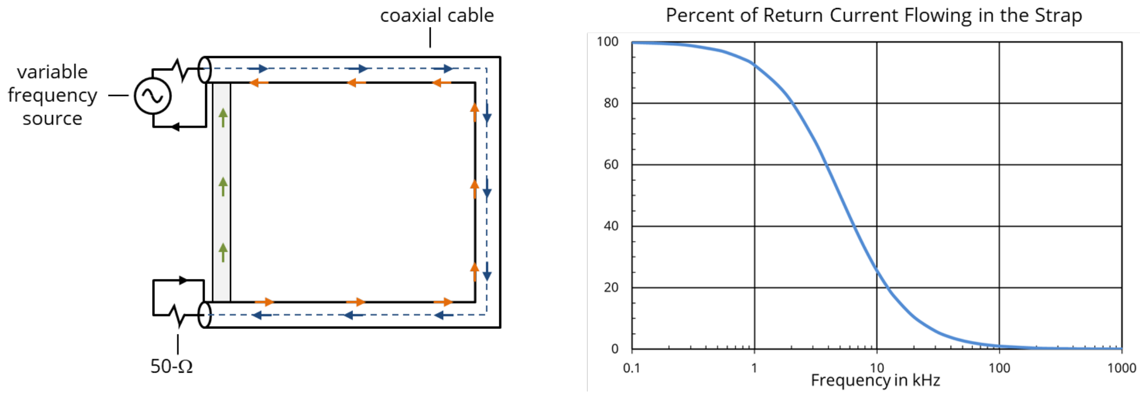


Figure 2.23. A simple current path demonstration.

The current will favor the lowest impedance path. The impedance of any electrically small current path is $R + j\omega L$, where R is the path resistance and L is the path inductance. At low frequencies, the impedance is primarily determined by the resistance. Since the shorting strap has a lower resistance than the longer cable shield, most of the current flows in the strap.

However, when current returns through the strap, the loop area of the current path is relatively large. At high frequencies, the inductance becomes a more important parameter than the resistance and the path of least impedance is the path of least inductance. Therefore, at high frequencies, current returns on the cable shield. This path minimizes the loop area and is therefore the path of least inductance.

In the example in Figure 2.23, the frequency at which the resistance and the inductive reactance are equal is about 5 kHz. The exact cut-off frequency will depend on the materials and the geometry of the path. However, for most practical circuit and system configurations, the path of least impedance will be the path of least resistance at kilohertz frequencies and lower. It will be the path of least inductance at megahertz frequencies and higher.

Consider the printed circuit board illustrated in Figure 2.24. Signal current from an output pin of Device 1 flows through a copper trace to an input pin of Device 2. We'll assume the current into Device 2 comes out the pin labeled "GND" and the current into Device 1 comes in the pin labeled "GND" and that both "GND" pins are connected to a solid copper plane on the board. What is the current return path in this situation?

Figure 2.25(a) illustrates the current distribution on a conducting plane under a microstrip trace when the current takes the path of least inductance. Note that most of the current returns in a band that is only a few trace heights wide. At megahertz frequencies and higher, the inductance will determine the current return path, and currents will flow on the ground plane primarily in a narrow path directly under the trace as indicated in Figure 2.26(a).

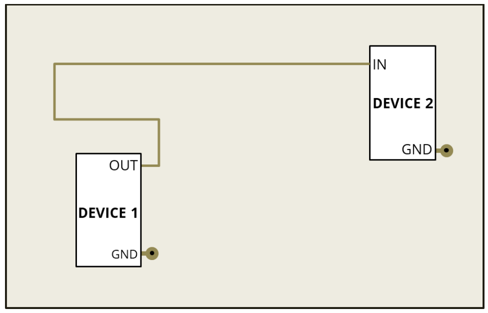


Figure 2.24. A simple printed circuit board with two components.

Figure 2.25(b) illustrates the current distribution on a plane when the resistance of the plane is the dominant contributor to the path impedance. The current density is essentially uniformly distributed across the plane and inversely proportional to the width of the plane.[5] Viewed from the top as illustrated in Figure 2.26(b), the current spreads out from the point where it is deposited on the plane and comes together again at the point where it leaves the plane. The low-frequency current distribution is independent of the signal trace routing path.

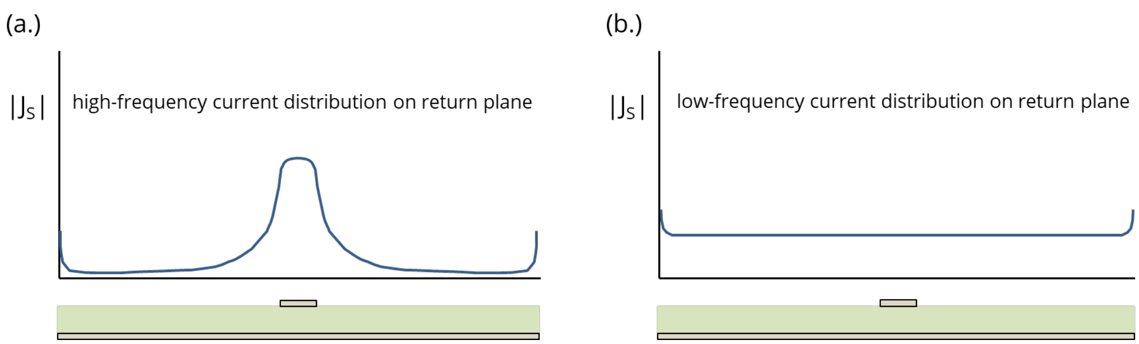


Figure 2.25. Current density on the surface of a plane beneath a microstrip trace a.) when the inductance dominates and b.) when the resistance dominates.

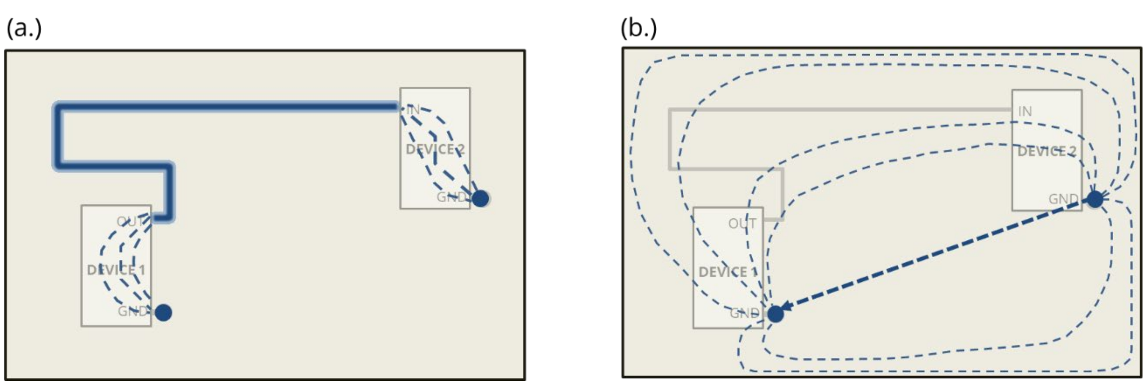
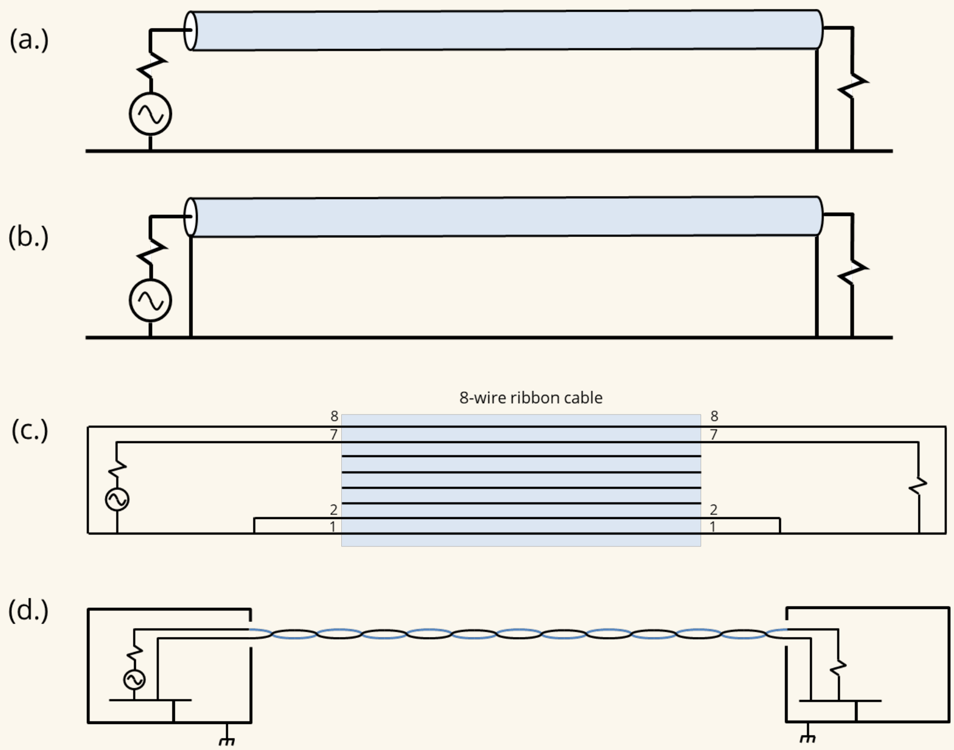


Figure 2.26. Current path on the plane of the board a.) at MHz frequencies and above and b.) at kHz frequencies and below.

[5] The low-frequency current density distribution is not perfectly flat. It varies with the distance of the current path between the source and the load as illustrated in Figure 2.26(b), becoming less uniform near the source and load points.

Example 2-11: Identifying Current Return Paths

For each of the configurations illustrated below, identify the primary return current paths.



In the first configuration above, there is only one possible path for the return current to take. Therefore, all low-frequency and high-frequency currents must return on the metal surface. In the second configuration, the cable shield grounded at both ends provides an alternative return path. Currents at megahertz frequencies and higher will return to the source on the shield of the coaxial cable. Kilohertz and lower frequency currents will distribute themselves between the two conductors based on their relative resistances.

For the ribbon cable configuration, low-frequency currents will return primarily on wires 1, 2 and 8 with an equal amount of current on each wire. High-frequency currents will return primarily on wire 8.

The last configuration illustrates two devices communicating with a twisted-wire pair. The signal current flows out one wire in the pair and, at high frequencies, returns on the other wire in the pair. However, at low frequencies, a significant fraction (perhaps most) of the current will return through the chassis grounds of each device. This unintended return path can result in a variety of EMC problems as we will see in the following sections.