# Electromagnetic Compatibility Course Notes

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# PREFACE

I first taught an EMC Course in 1989 at North Carolina State University. The course text was Henry Ott's, *Noise Reduction Techniques in Electronic Systems, 2nd ed.* At the time, I was an EMC engineer working for IBM. I had met Mr. Ott and received a copy of his book when he visited IBM to teach an EMC short course.

A few years later, I joined the faculty at the University of Missouri-Rolla where I met Dr. Tom Van Doren. Dr. Van Doren had been teaching an EMC course at the university called *Grounding and Shielding* since the mid-1980s. In 1992, he graciously allowed me to take over as the course instructor. I adopted Dr. Clayton Paul's newly released book, *Introduction to Electromagnetic Compatibility*, as the course text. Dr. Paul's book was structured like a university course text. It was mathematically rigorous and stressed fundamental concepts. It was an excellent fit for the *Grounding and Shielding* course, but it did not cover some of the topics that were an important part of Dr. Van Doren's course. I supplemented the material in the text with "course notes" that I wrote myself and passed out to the students. Each year, the course notes expanded. Eventually, the textbook became optional, and the course was taught primarily from the course notes.

The volume and scope of the notes continued to grow as topics were added and updated. Material was also deleted each year as it became outdated. I continued to use and revise the course notes for my EMC classes after I moved to Clemson University in 2006. And after retiring from Clemson, I started distributing the notes to students attending my EMC short courses. Over the years, the content was adapted to suit this new audience.

The course notes are intended to serve as a textbook for a university course in electromagnetic compatibility. However, I hope that they will also be helpful self-study guides for practicing engineers who have never had an opportunity to take a university-level EMC course.

Updates and corrections will be posted on the LearnEMC.com website, where solutions to the problems in this book will also be available.

I want to thank six people who reviewed an early draft of this book and provided critical feedback that helped to make these notes more comprehensive and valuable to a broader audience.

Dr. Thomas Van Doren
Dr. Daryl Beetner
Dr. Frank Leferink
Mr. Reto Keller
Mr. Ken Wyatt
Dr. Min Zhang

Each of these respected members of the EMC community made valuable suggestions that helped to clarify and expand these notes.

Todd Hubing

## Using These Course Notes

Topics are organized in the order they would typically be taught in a university EMC course. Later chapters build upon the content covered in the earlier chapters. Generally, the content is formatted and presented as it would be in a standard engineering textbook. However, the reader should be aware of the following non-standard features.

1. In Chapter 2, and whenever new fundamental quantities are introduced, equations for determining these quantities are followed by the mks units that these quantities would typically have (e.g., $C = \frac{Q}{V} \text{ farads}$). While these equations are valid for any consistent set of units, the course notes provide the appropriate mks units to help students relate these quantities to familiar circuit and field parameters.
2. Each chapter has one or more *Quiz Questions* outlined in a rectangular box. These are relatively simple questions that are answered in the text shortly after the question statement. The purpose is to illustrate fundamental questions an EMC engineer might need to answer and encourage the reader to think about them before an answer is provided.
3. Each chapter also has one or more *Example* problems. These examples demonstrate how concepts and equations in the text are applied to the solution of actual problems of interest.
4. Calculations in this book are generally rounded to an appropriate number of significant figures. However, occasionally an extra significant figure is included to remove any ambiguity as to how the calculation was performed.
5. There are *Suggested References* at the end of each chapter, along with a brief explanation of why they are suggested. These are general references related to the content of the chapter. Specific references to sources of equations or facts appear as footnotes at the bottom of the appropriate page.
6. *Problems* are provided at the end of each chapter that demonstrate practical applications of the course material. As in most university engineering courses, the real learning takes place when students are asked to apply the course material to problems and situations that are a little different from the problems solved for them in the classroom. Answers will be available on the LearnEMC.com website; however, students are strongly encouraged to try to solve each problem before looking up the answer.
7. Errors and corrections to the material in these course notes will be posted on the LearnEMC.com website. If you find errors that are not reported on the website, please let us know by emailing *publications@learnemc.com*.

## CHAPTER 1

# INTRODUCTION TO ELECTROMAGNETIC COMPATIBILITY

Electromagnetic compatibility (EMC) is broadly defined as the state that exists when all electronic devices in a system function without error in their intended electromagnetic environment. If a device generates electromagnetic fields or currents that interfere with the operation of another device, there is an electromagnetic compatibility issue. If an electronic product works great in the laboratory, but fails at the customer location due to electrostatic discharge, lightning, or other forms of electromagnetic interference (EMI), the product has an electromagnetic compatibility problem.

Virtually all electronic products are subjected to EMC testing during development and must meet certain EMC requirements before being marketed or sold. The electromagnetic noise emanating from these products is carefully measured and regulated. Many products are also subjected to strong external fields or voltage transients to ensure they can operate normally in noisy environments.

To a knowledgeable design engineer, meeting EMC requirements is no different than meeting functional, thermal, or safety requirements. It’s simply a matter of being aware of the requirements and the steps necessary to ensure compliance. With a well-thought-out design, most products have no trouble meeting their EMC requirements. However, while most product designers understand the importance of addressing functional, thermal and safety requirements early in the development cycle, it’s not uncommon for the first attempts to address EMC issues to occur after a product fails its first compliance test. At that point, design options tend to be limited. EMC “fixes” can (and often do) delay development schedules and increase product cost.

Many companies, especially those with short product development cycles, recognize the importance of addressing EMC early in the design process. They rely on engineers who understand what it takes to design EMC-compliant products. For these companies, EMC testing is a formality. Their products tend to be more reliable and lower-cost than similar products that had to be “fixed” after failing an EMC test.

As one might expect, engineers who understand electromagnetic compatibility principles and can apply them to product design are in high demand. They possess a skill set that includes a basic understanding of electromagnetic field theory, as well as a familiarity with real-world circuits, components, and systems. They understand the importance of electrical ground and recognize that ground and current return are different concepts. They know that the physical layout of a system is important, and that circuit behavior is often governed by component interactions that do not appear in circuit schematics. Basically, the engineers who understand what it takes to design EMC-compliant products understand the concepts described in these course notes and have learned to apply them to real-world product designs.

## 1.1. Elements of an EMC Problem

There are three essential elements to an EMC problem, as illustrated in Figure 1.1. There must be a source of electromagnetic energy, a receptor (or victim) that cannot function properly due to interference from the source, and a path that couples the interfering energy from the source to the receptor. Each of these three elements must be present for the interference to occur. Electromagnetic compatibility problems are generally solved by identifying at least two of these elements and eliminating (or attenuating) one of them.

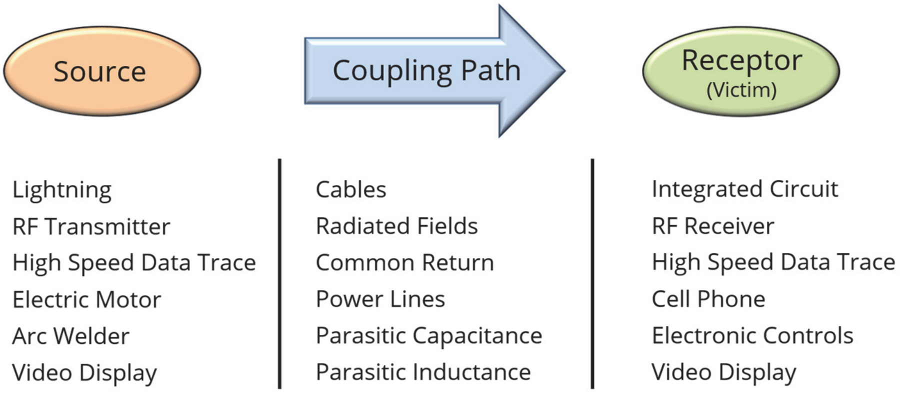

**Figure 1.1.** The three essential elements of an EMC problem.

For example, there have been several documented cases of EMI causing nuclear power plants to go into an emergency shutdown.[1] In at least one case, the noise was a spurious voltage spike in a ventilation flow sensor circuit. In another case, it was a neutron flux sensor. The first element identified in both cases was the victim circuit (a sensor). In both cases, the last leg of the coupling path was conducted noise on the sensor cable.

The apparent source of the flow sensor malfunction was the keying of wireless handsets operated by plant personnel. The source of the flux sensor interference was noise generated by the actuation of relays in another circuit. Since it’s only necessary to eliminate one of the three elements, it was possible to solve the problem in both cases with noise suppression on the sensor cable.

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[1] United States Nuclear Regulatory Commission public domain event reports available on the U.S.NRC website at nrc.gov.

## Suggested References

1. IEEE Transactions on Electromagnetic Compatibility. IEEE, Piscataway, New Jersey, USA. *Peer-reviewed research results on a wide variety of topics related to electromagnetic compatibility.*
2. Electromagnetic Compatibility Magazine, IEEE, Piscataway, New Jersey, USA. *News, tutorials, and technical articles related to developments in the field of electromagnetic compatibility.*
3. Banana Skins, Nutwood UK Ltd. *Descriptions of electromagnetic interference events collected from anecdotes, reports, official documents and news stories.*

## Problems

1. For each of the EMC problems below, identify the probable source, coupling path and receptor.

   a.) A computer display turns blue when a microwave oven is operating.

   b.) Radio station presets are erased when the car is started.

   c.) A cable modem stops working during a thunderstorm and won’t power up anymore.

   d.) WIFI connections are dropped whenever an electric blanket is set on high.

   e.) Your car signals that it has low tire pressure every time you drive by the airport.

2. What are alternative names for conducted, inductive, and capacitive coupling?
3. At a given frequency, we can convert an rms voltage to a peak voltage by multiplying the rms value by $\sqrt{2}$. If the rms voltage is expressed in dB(μV), how many dB must be added to get the peak voltage expressed in dB(μV)?
4. Express 150 mV in dB(mV), dB(μV) and dBm (as displayed on a 50-Ω spectrum analyzer display.
5. The ratio, 50 $V_{rms}$ to 25 $V_{rms}$, is a factor of 2 or 6 dB. What is the ratio of 50 $V_{peak}$ to 25 $V_{peak}$ expressed in dB?
6. A 1-volt signal passes through an amplifier with 12 dB of gain, then through an attenuator with 6 dB of loss. What is the amplitude of the signal coming out of the attenuator?
7. A 1-volt signal passes through an amplifier with 50% gain. What is the amplitude of the output? Repeat your calculation for amplifiers with gains of 100%, 200% and 500%.
8. A measured field strength is 46 dB(μV/m), but the limit is 40 dB(μV/m). By how much does the measured field strength exceed the limit?

## CHAPTER 4

# SIGNAL SPECTRA

Conducted and radiated emissions limits are specified as a function of frequency. Many immunity tests are performed using swept-frequency sources. Designing for electromagnetic compatibility requires a basic understanding of how signals that are functions of time contain power at various frequencies. It's also important to understand how noise with a given frequency content will impact a signal that is a function of time.

## 4.1. Time and Frequency Domains

Electrical signals have both time- and frequency-domain representations. In the time domain, voltage or current is expressed as a function of time, as illustrated in Figure 4.1. Most engineers are relatively comfortable with time-domain representations of signals. Signals measured on an oscilloscope are displayed in the time domain, and digital information is often conveyed by a voltage as a function of time.

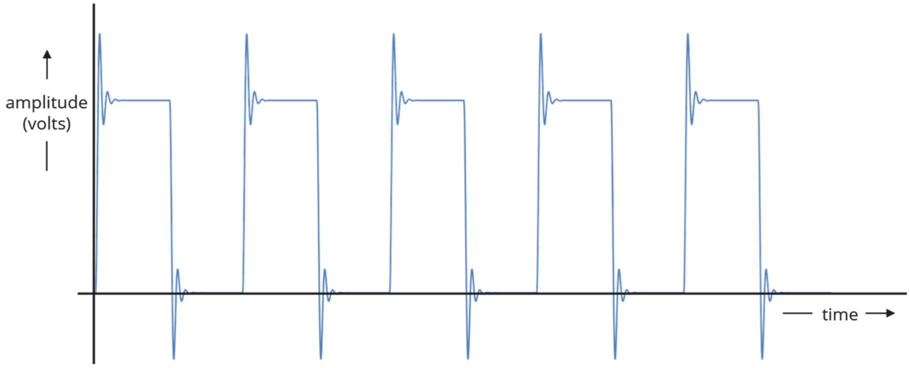


**Figure 4.1.** Time-domain representation of an electrical signal.

Signals can also be represented by their magnitude and phase as functions of frequency. Signals that repeat periodically in time every $T$ seconds are represented by a line spectrum, as illustrated in Figure 4.2. The line spectrum has a DC component at 0 Hz, a fundamental component at $1/T$, and harmonics at $n/T$ (where $n$ is an integer). This representation is also called a power spectrum because the sum of the powers in each harmonic equals the time-average power in the time-domain signal.

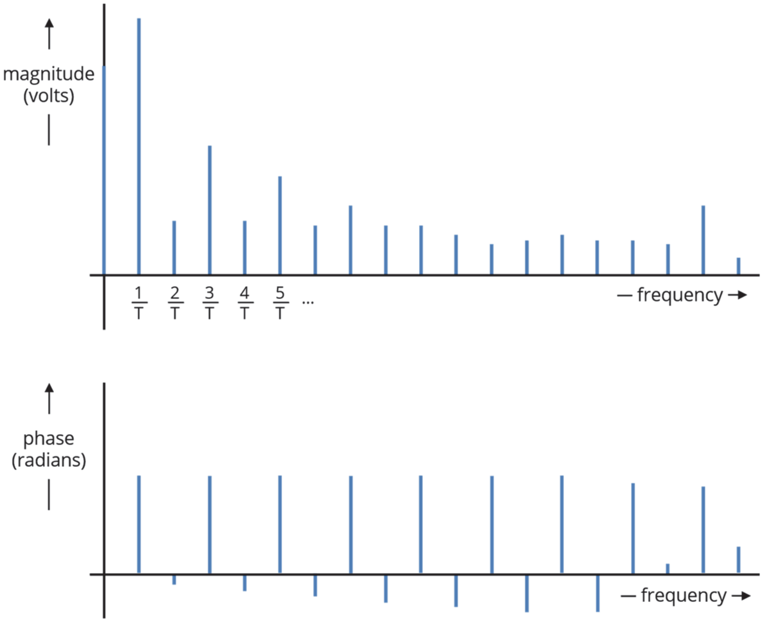


**Figure 4.2.** Frequency-domain representation of a periodic signal.

Signals that are time-limited (i.e., are only non-zero for a finite time) are represented by a continuous spectrum as illustrated in Figure 4.3. This representation is also referred to as an *energy spectrum* because the integral of the energy density in this waveform over frequency equals the total energy in the time-domain signal.

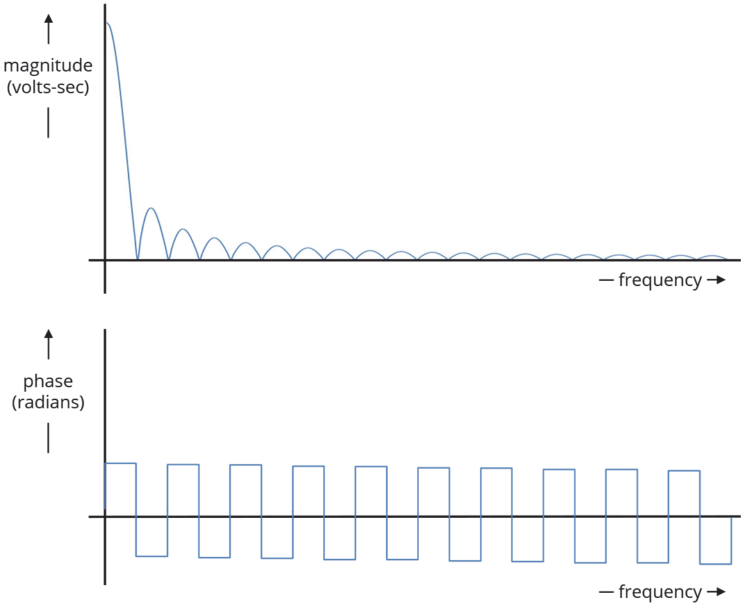


**Figure 4.3.** Frequency-domain representation of a time-limited signal.

Frequency-domain representations are particularly useful for analyzing linear systems. EMC and signal integrity engineers must be able to work with signals represented in both the time and frequency domains. Signal sources and interference are often defined in the time domain. However, system behavior and signal transformations are more convenient and intuitive when working in the frequency domain.

## 4.2. Linear Systems

Linear system theory plays a key role in the engineering analysis of electrical and mechanical systems. Engineers model a wide variety of physical systems as linear transformations, including circuit behavior, signal propagation, coupling and radiation. Therefore, it is important to clarify exactly what we mean by a linear system so that we recognize when and how to use the powerful linear system analysis tools available to us.

Figure 4.4 illustrates a system with one input, $x(t)$, and one output, $y(t)=H\{x(t)\}$. If an input $x_1(t)$ produces an output $y_1(t)$, and an input $x_2(t)$ produces an output $y_2(t)$, then the system is linear if and only if,

$$a\,y_1(t)+b\,y_2(t)=H\left\{a\,x_1(t)+b\,x_2(t)\right\} \tag{4.1}$$

where $a$ and $b$ are constants. In other words, scaling the input by a constant will produce an output scaled by the same constant, and combining (summing) two inputs will produce an output that is the sum of the outputs produced by each individual input.

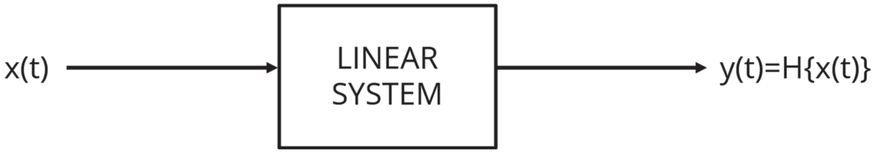


**Figure 4.4.** A linear system with input x(t) and output y(t).

### Quiz Question

*Which of the following equations describes the relationship between the output y(t) and the input x(t) of a linear system?*

- a.) $y=5x$
- b.) $y(t)=0$
- c.) $y=8x+3$
- d.) $y=x^2$
- e.) $y(t)=5t\,x(t)$
- f.) $y=\sin x$
- g.) $y(t)=5\frac{\partial}{\partial t}[x(t)]$

Of the choices presented in this question, only *a*, *b* and *g* are linear system transformations. $y(t)=0$ is not a very interesting system, because its output is always zero, but it is linear. Simple derivative and integral operators are linear because they satisfy the conditions in Equation (4.1). The remaining choices are not linear operations. Note that $y=8x+3$ is the equation of a straight line, but it does not describe a linear system because it has a non-zero output when there is no input.

At first, it may appear that very few real electrical or mechanical systems of interest behave this way. However, many non-linear systems can be approximated as linear over a limited subset of possible input values. Most engineering analysis depends on modeling real devices and circuits as linear systems.

## 4.3. Frequency-Domain Analysis of Linear Systems

Linear systems have the unique property that any sinusoidal input will produce a sinusoidal output at exactly the same frequency. In other words, if the input is of the form,

$$x(t) = A_{in} \cos(\omega_0 t + \phi_{in}), \tag{4.2}$$

then the output will have the form,

$$y(t) = A_{out} \cos(\omega_0 t + \phi_{out}). \tag{4.3}$$

In general, the magnitude and phase of the sinusoidal signal may change, but the frequency must be constant. This provides us with a very powerful analysis tool for analyzing linear systems. If we represent an input signal as the sum of its components in the frequency domain, then we can express the output as a simple scaling of the component magnitudes and shifting of the component phases.

## 4.4. Phasor Notation

To facilitate the analysis of linear system responses to sinusoidal inputs, it is convenient to represent signals in an abbreviated form known as *phasor notation*. Consider an input of the form,

$$x(t) = A_{in} \cos(\omega t + \phi_{in}), \tag{4.4}$$

This can be represented as,

$$\begin{aligned} x(t) &= \mathrm{Re}\left\{A e^{j(\omega t+\varphi)}\right\} \\ &= A \cdot \mathrm{Re}\left\{e^{j\omega t} e^{j\varphi}\right\} \end{aligned} \tag{4.5}$$

where $\mathrm{Re}\{\bullet\}$ indicates the real part of a complex quantity. Recognizing that the frequency $\omega$ will be the same throughout the system, we don't need to write the term $e^{j\omega t}$ explicitly, as long as we remember that it's there. The same applies to the $\mathrm{Re}\{\bullet\}$ notation. This allows us to express a sinusoidal signal simply in terms of its magnitude and phase as,

$$x = Ae^{j\phi} \quad \text{or} \quad A\angle\phi\,. \tag{4.6}$$

The expression in (4.6) is the signal in (4.4) represented using phasor notation. Note that we must know the frequency of a signal in order to convert from phasor notation to the time-domain representation.

### Quiz Question

*Write the following signals using phasor notation:*

*a.) x(t) = 5 cos(ωt) volts*
*b.) y(t)=5 sin(ωt) amps*
*c.) z(t) = 5t sin(ωt) volts*

The first signal expressed in phasor notation is simply $x$ = 5 volts. To obtain the phasor notation for the second signal, we recognize that sin(ωt) = cos(ωt + π/2), so $y = e^{j\pi/2}$ amps. The third signal is not a sinusoid and therefore cannot be expressed using phasor notation.

## 4.5. Fourier Series

Of course, many of the inputs to linear systems that we would like to analyze are not sinusoidal. In this case, it is desirable to represent time-domain signal waveforms as a sum of sinusoidal frequency components. In the frequency domain, each component can be analyzed individually. The frequency-domain system outputs can then be summed and converted back to the time domain.

A periodic signal can be represented as a sum of its frequency components by calculating its *Fourier series* coefficients. A periodic signal with period $T$ can be written

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn2\pi f_0 t} \tag{4.7a}$$

where $f_0 = 1/T$ and

$$c_n = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)\, e^{-jn2\pi f_0 t}\, dt\,. \tag{4.7b}$$

If $x(t)$ is a real time-domain signal, the coefficients $c_n$ and $c_{-n}$ are complex conjugates (i.e., $c_{-n} = c_n^*$ ) and we can rewrite Eq. (4.7a) in the form,

$$\begin{aligned} x(t) &= c_0 + \sum_{n=1}^{\infty}\left(c_n e^{jn2\pi f_0 t} + c_n^* e^{-jn2\pi f_0 t}\right) \\ &= c_0 + \sum_{n=1}^{\infty}\left(\left|c_n\right| e^{jn2\pi f_0 t + \phi_n} + \left|c_n\right| e^{-\left(jn2\pi f_0 t + \phi_n\right)}\right) \\ &= c_0 + \sum_{n=1}^{\infty} 2\left|c_n\right| \cos\left(n2\pi f_0 t + \phi_n\right). \end{aligned} \tag{4.8}$$

The Fourier series coefficients in this form consist of a DC component, $c_0$, and positive harmonic frequencies, $n2\pi f_0$ ($n$ = 1,2,3, …). This is the *one-sided Fourier series* and the coefficients, $2\left|c_n\right|$, represent the peak value of each harmonic. Dividing the peak value by $\sqrt{2}$ yields the root-mean-square (rms) value. Signal harmonics measured with a spectrum analyzer or EMI test receiver are the rms values of the one-sided Fourier Series coefficients. In other words, the amplitude of each measured harmonic is $\sqrt{2}\left|c_n\right|$.

The frequency-domain representation of a periodic signal is a line spectrum. It can only have non-zero values at DC, the fundamental frequency, and harmonics of the fundamental. Because periodic signals have no beginning or end, non-zero periodic signals have infinite energy but finite power. The total power in the time-domain signal,

$$P_{total} = \frac{1}{T}\int_{t_0}^{t_0+T} x^2\left(t\right) dt \tag{4.9}$$

is equal to the sum of the power in each frequency-domain component,

$$P_{total} = \sum_{n=-\infty}^{\infty} \left|c_n\right|^2 . \tag{4.10}$$

A few periodic signals and their frequency-domain representations are illustrated in Figure 4.5.

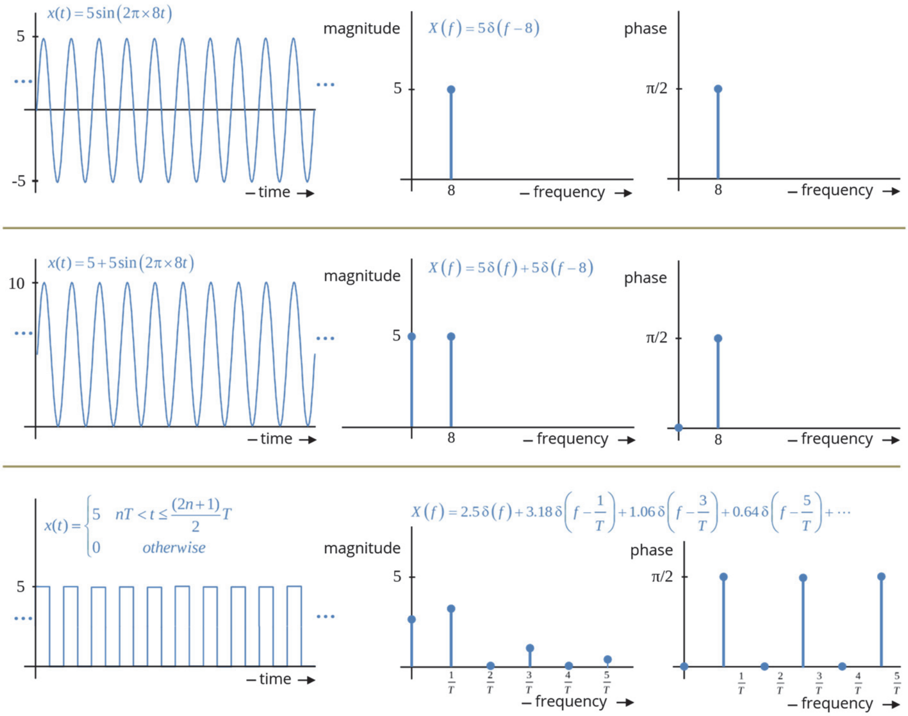
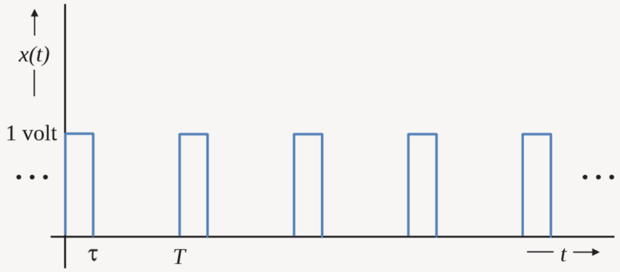


**Figure 4.5.** Periodic signals in the time and frequency domains.

### Example 4-1: Frequency-Domain Representation of a Pulse Train

*Determine the frequency-domain representation for the pulse train shown in the figure below.*



In the time domain, this signal is described by the following formula:

$$x(t)=\begin{cases}1\text{ V} & nT<t<nT+\tau\\ 0 & \textit{otherwise}\end{cases}\qquad n=\pm1,\pm2,\pm3,\cdots.$$

The coefficients of the Fourier series are then calculated using Eq. (4.7b) as,

$$\begin{aligned}c_n&=\frac{1}{T}\int_0^T x(t)\,e^{-jn2\pi f_0 t}dt\\ &=\frac{1}{T}\int_0^\tau (A)\,e^{-jn2\pi t/T}dt\\ &=\frac{A}{T}\int_0^\tau e^{-jn2\pi t/T}\,dt\\ &=\frac{A\tau}{T}\left[\frac{\sin\left(n\pi\tau/T\right)}{\left(n\pi\tau/T\right)}\right]e^{-j\left(n\pi\tau/T\right)}.\end{aligned}$$

Note that as $\tau\to 0$, our time-domain signal looks like an impulse train, and the amplitudes of all the harmonics approach the same value. As $\tau\to\frac{T}{2}$, the signal becomes a square wave, and the magnitude of the harmonics becomes,

$$c_n=\frac{A}{2}\left|\frac{\sin\left(n\pi/2\right)}{\left(n\pi/2\right)}\right|\left|e^{-j\left(n\pi/2\right)}\right|=\begin{cases}\dfrac{A}{n\pi} & n=\pm1,\pm3,\pm5\cdots\\ 0 & n=\pm2,\pm4,\pm6\cdots\end{cases}.$$

In this case, the amplitude of the even harmonics is zero, and the odd harmonics decrease linearly with frequency ($n$).

Note that if we wanted to determine the amplitude of the harmonics as measured on a spectrum analyzer, we would calculate the rms amplitude of the one-sided Fourier Series coefficients,

$$\sqrt{2}\left|c_n\right|=\frac{\sqrt{2}A\tau}{T}\left[\frac{\sin\left(n\pi\tau/T\right)}{\left(n\pi\tau/T\right)}\right]\qquad n=1,2,3,\cdots$$

## 4.6. Fourier Transform

Transient signals (i.e., signals that start and end at specific times) can also be represented in the frequency domain using the *Fourier transform*. The Fourier transform representation of a transient signal, *x(t)*, is given by,

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \,. \tag{4.11}$$

The inverse Fourier transform can be used to convert the frequency-domain representation of a signal back to the time domain,

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df \,. \tag{4.12}$$

Two transient time-domain signals and their Fourier transforms are illustrated in Figure 4.6.

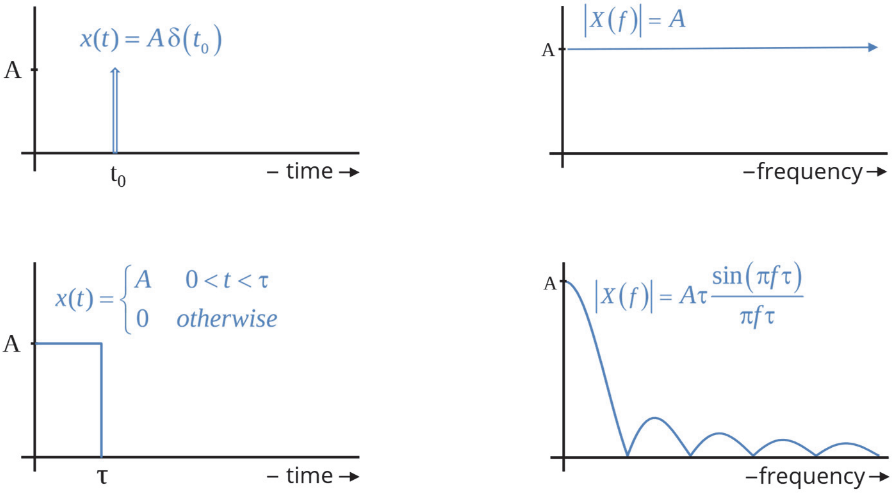


**Figure 4.6.** Transient signals in the time and frequency domain.

Note that transient signals have zero average power (when averaged over all time), but they have finite energy. The total energy in a transient time-domain signal is given by,

$$\mathrm{E} = \int_{-\infty}^{\infty} x^2(t)\, dt \,. \tag{4.13}$$

This must equal the total energy in the frequency-domain representation of the signal,

$$\mathrm{E} = \int_{-\infty}^{\infty} |X(f)|^2 \, df \,. \tag{4.14}$$

## 4.7. Trapezoidal Signal Waveforms

Let’s examine the frequency-domain representation of the periodic trapezoidal waveform illustrated in Figure 4.7. Examining this waveform's behavior helps

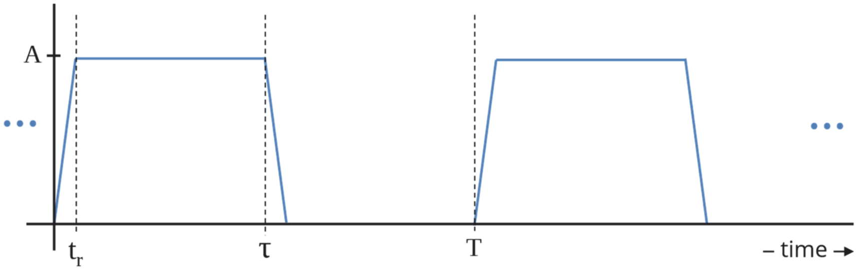


**Figure 4.7.** Trapezoidal waveform.

us gain insight into the relationship between time- and frequency-domain representations in general. Also, the similarity between the trapezoidal waveform and common digital signal waveforms will be useful when investigating EMC or signal integrity issues in digital systems.

Using the one-sided Fourier series, Equations (4.7b) and (4.8), we can represent this signal as the sum of its frequency components,

$$x(t) = c_0 + \sum_{n=1}^{\infty} 2\left|c_n\right| \cos\left(n 2\pi f_0 t + \phi_n\right) \tag{4.15}$$

where

$$2\left|c_n\right| = \frac{2A\tau}{T}\left|\frac{\sin\left(n\pi\tau / T\right)}{\left(n\pi\tau / T\right)}\right|\left|\frac{\sin\left(n\pi t_r / T\right)}{\left(n\pi t_r / T\right)}\right|. \tag{4.16}$$

Equation (4.16) can be derived by noting that the trapezoidal waveform in Figure 4.7 can be obtained by convolving the pulse train in Example 4-1 with another pulse train whose pulses have a width, $t_r$, and an amplitude $A/t_r$. Convolution in the time domain is equivalent to multiplication in the frequency domain, so we can multiply the frequency-domain representations of these pulse trains to obtain Eq. (4.16).

Each term, $2|c_n|$, is the peak amplitude of the nth harmonic. If we assume that $t_r \ll T$, we note that the third term is $\frac{\sin\left(\text{small number}\right)}{\text{small number}} \approx 1$ for the lower harmonics. If $\tau = T/2$ (i.e., a 50% duty cycle), then the numerator of the second term is 1 for the harmonics (n = 1,3,5…) and 0 for the even harmonics (n = 2,4,6…). The amplitude of the lower harmonics is then inversely proportional to n (i.e., the amplitude of the lower harmonics decreases proportionally to the frequency). At higher harmonics, the third term also begins to decrease proportionally to frequency, so the overall amplitude of the upper harmonics decreases on average at a rate proportional to the square of the frequency.

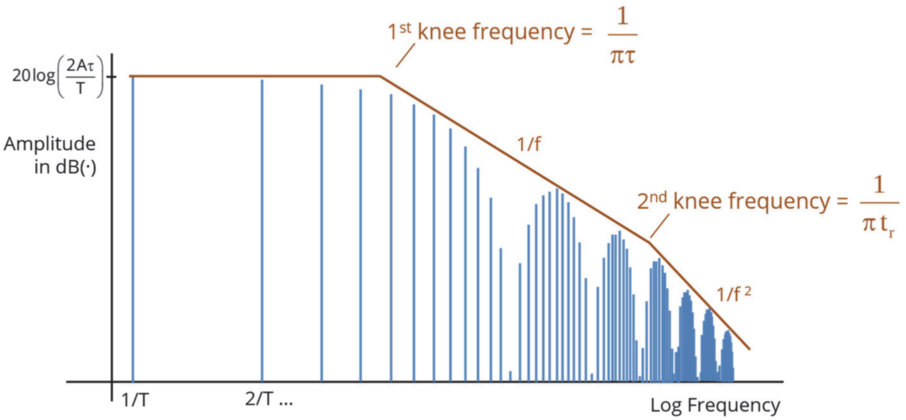


**Figure 4.8.** Frequency-domain representation of a trapezoidal signal $\left(\tau < T/2, t_r \ll T\right)$.

The frequency-domain representation of a trapezoidal pulse train $\left(\tau < T/2, t_r \ll T\right)$ and its envelope are illustrated in Figure 4.8. Note that small values of $\tau$ (short duty cycles) reduce the amplitude of the harmonics below the first knee frequency. These lower harmonics then have approximately the same amplitude.

For most EMC calculations, we want to know the rms amplitude of the harmonics, not the peak amplitude. This is easily found by dividing the amplitude of each non-zero harmonic by the square root of 2. Also, the equation for the envelope is generally more useful than Equation (4.16), because nulls in the pattern can move around as the duty cycle changes. EMC modeling generally requires knowing the maximum amplitude each harmonic can attain. The equation for the envelope of the rms harmonic amplitudes is,

$$V_{rms-\max}(f) = \begin{cases} \dfrac{\sqrt{2}A\tau}{T} & \text{when } f < \dfrac{1}{\pi\tau} \\ \dfrac{\sqrt{2}A}{\pi}\left(\dfrac{f_0}{f}\right) & \dfrac{1}{\pi\tau} < f < \dfrac{1}{\pi t_r} \\ \dfrac{\sqrt{2}A}{\pi^2}\left(\dfrac{f_0}{f}\right)\left(\dfrac{1}{t_r f}\right) & \text{when } f > \dfrac{1}{\pi t_r} \end{cases} . \quad (4.17)$$

This equation assumes that $0 \le \tau \le \frac{T}{2}$ (i.e., the duty cycle is 50% or less). For duty cycles greater than 50%, we can redefine $\tau$ as the *off* time rather than the *on* time.

Equation (4.17) is easier to calculate than (4.16), but in many cases, we can avoid using a calculator at all by noting that the amplitude of the fundamental

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